Partial Fractions

Degrees of Divisor

Linear

$f(x) = ax^3 + bx^2 + cx + d$

divisor $ax+b$

$$ax+b \;\overline{)\; ax^3+bx^2+cx+d}$$

$ax^3 + bx^2$

$x^2 + p$

$cx + d$

$cx + q$

r ← deg 0

$ax+b$ ← deg 1

$$\frac{f(x)}{ax+b} = x^2 + p + \frac{r}{ax+b}$$

Quadratic Divisor

$f(x) = x^3 + bx^2 + cx + d$

divisor $(x+k)^2$

$$(x+k)^2 \;\overline{)\; x^3+bx^2+cx+d}$$

$x+1$

$x^3 + 2kx^2 + k^2x$

$x^2 + nx + d$

$x^2 + 2kx + k^2$

$px + q$

$$\frac{f(x)}{(x+k)^2} = x+1 + \frac{px+q}{(x+k)^2}$$

$px+q$ ← incomplete!

$(x+k)^2$ ← split into linear and quadratic forms

Further Partial

$$\frac{px+q}{(x+k)^2} = \frac{r}{x+k} + \frac{s}{(x+k)^2}$$

deg 1

recombining gives deg 0 or 1

Cannot be factorized

$f(x) = ax^3 + bx^2 + cx + d$

divisor ax^2+b ← cannot be factorized

$$ax^2+b \;\overline{)\; ax^3+bx^2+cx+d}$$

$x + n$

$ax^3 + bx$

$bx^2 + px + d$

$bx^2 + h$

$px + q$ ← deg 1

$$\frac{f(x)}{ax^2+b} = x+n + \frac{px+q}{ax^2+b}$$

deg 1