2016 N(A) AM P1 5 Oct 2016

1. $\frac{d}{dx}\, 2x^3\sqrt{1-x}$

$= \frac{1}{2}(1-x)^{-\frac{1}{2}}(-1)(2x^3) + 6x^2\sqrt{1-x}$

$= x^2(1-x)^{-\frac{1}{2}}\left[-x + 6(1-x)\right]$

$= \dfrac{x^2(6-7x)}{\sqrt{1-x}}$

2. 4th term, when $r=3$,

4th term $= \binom{6}{3}(a)^{6-3}(-x)^3$

$= -20a^3x^3$

$\Rightarrow -20a^3 = -540$

$a^3 = 27$

$a = 3$

3. $2x^2 + \frac{x}{k} - 8 = 0$

For equation to have equal roots, let discriminant $= 0$.

$\left(\frac{1}{k}\right)^2 - 4(2)(-8) = 0$

$\frac{1}{k^2} + 64 = 0$

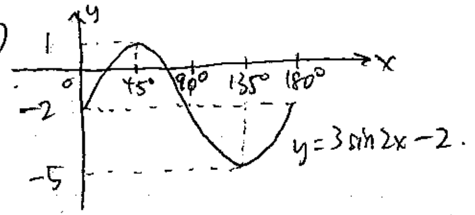
$k^2 = -\frac{1}{64}$

Since $k^2 > 0$, there are no real k for discriminant $= 0$.

4 (i) period $= 180°$

Amplitude $= 3$

4(ii)



5 (i) $\int (x-2)^{19}\, dx$

$= \dfrac{(x-2)^{20}}{20} + c$

5 (ii) $\int \dfrac{x^4+1}{x^4}\, dx$

$= \int 1 + \frac{1}{x^4}\, dx$

$= x + \dfrac{x^{-3}}{-3} + c$

$= x - \dfrac{1}{3x^3} + c$

6. $3x - 5\sqrt{2} = x\sqrt{2} + 4$

$3x - x\sqrt{2} = 5\sqrt{2} + 4$

$x(3-\sqrt{2}) = 5\sqrt{2} + 4$

$x = \dfrac{5\sqrt{2}+4}{3-\sqrt{2}} \times \dfrac{3+\sqrt{2}}{3+\sqrt{2}}$

$= \dfrac{15\sqrt{2} + 10 + 12 + 4\sqrt{2}}{9-2}$

$= \frac{19}{7}\sqrt{2} + \frac{22}{7}$

where $p = \frac{22}{7}$, $q = \frac{19}{7}$

7. $V = \frac{2}{3}\pi z^3$

$\frac{dV}{dz} = 2\pi z^2$

When $z = 0.8$, $\frac{dV}{dz} = \frac{32}{25}\pi$.

$\frac{dV}{dt} = \frac{dV}{dz} \times \frac{dz}{dt}$

$0.5 = \frac{32}{25}\pi \times \frac{dz}{dt}$

$\frac{dz}{dt} = \frac{25}{64\pi}$ or 0.124 m/s (3sf.)

8 (i) let $3x - 1 = 0$

$x = \frac{1}{3}$

$f(\tfrac{1}{3}) = 3(\tfrac{1}{3})^3 - 7(\tfrac{1}{3})^2 - 22(\tfrac{1}{3}) + 8$

$= 0$

$\Rightarrow 3x-1$ is a factor of $f(x)$ (shown)

8(ii)

$$
\begin{array}{r}
x^2 - 2x - 8 \\
3x-1 \overline{\big)\, 3x^3 - 7x^2 - 22x + 8} \\
-(3x^3 - x^2) \\
\hline
-6x^2 - 22x \\
-(-6x^2 + 2x) \\
\hline
-24x + 8 \\
-24x + 8 \\
\hline
0
\end{array}
$$

$f(x) = 3x^3 - 7x^2 - 22x + 8$

$= (3x-1)(x^2-2x-8)$

$= (3x-1)(x+2)(x-4)$

9. $4\sin^2 x - 3 = \cos x$

$4(1-\cos^2 x) - 3 = \cos x$

$4\cos^2 x + \cos x - 1 = 0$

$\cos x = \dfrac{-1+\sqrt{17}}{8}$ or $\dfrac{-1-\sqrt{17}}{8}$

$= 0.3904$ (4sf.) $\quad = -0.6404$

basic $x = 67.02$ (2d.p.) $\quad$ basic $x = 50.18°$ (2d.p.)

$x = 67.02°, 360° - 67.02°$ $\quad$ $x = 180 - 50.18°, 180° + 50.18°$

$x = 67.0°, 129.8°, 230.2°, 293.0°$ (1d.p.)

10 (i) $5x^2+2mx+m-1=0$.

$\alpha+\beta=-\frac{2m}{5}$

$\alpha\beta=\frac{m-1}{5}$.

10 (ii) $\alpha^2+\beta^2=\frac{6}{25}$.

$(\alpha+\beta)^2-2\alpha\beta=\frac{6}{25}$.

$\left(-\frac{2m}{5}\right)^2-2\left(\frac{m-1}{5}\right)=\frac{6}{25}$.

$\frac{4m^2}{25}-\frac{2m-2}{5}=\frac{6}{25}$

$4m^2-10m+10=6$.

$4m^2-10m+4=0$.

$2m^2-5m+2=0$.

$(2m-1)(m-2)=0$.

$m=\frac{1}{2}$ or 2.

11. (i) $x^2+y^2-4x+6y=12$.

$(x-2)^2+(y+3)^2=12+2^2+3^2$

$(x-2)^2+(y+3)^2=25$

radius $=\sqrt{25}=5$ units.

centre, C is $(2,-3)$.

11. (ii) $a=$ radius $=5$.

$OP=a-\sqrt{b}$.

$PC-OC=5-\sqrt{b}$.

$5-\sqrt{2^2+3^2}=5-\sqrt{b}$.

$\Rightarrow b=2^2+3^2$

$=4+9$

$=13$.

12. (i) $\cos\left(\theta+\frac{\pi}{2}\right)$

$=\cos\theta\cos\frac{\pi}{2}-\sin\theta\sin\frac{\pi}{2}$

$=-\sin\theta$.

$\sin\left(\theta-\frac{\pi}{6}\right)$

$=\sin\theta\cos\frac{\pi}{6}-\cos\theta\sin\frac{\pi}{6}$.

$=\sin\theta\left(\frac{\sqrt{3}}{2}\right)-\cos\theta\left(\frac{1}{2}\right)$.

$=\frac{\sqrt{3}\sin\theta-\cos\theta}{2}$.

12 (ii) $3\cos\left(\theta+\frac{\pi}{2}\right)=\sin\left(\theta-\frac{\pi}{6}\right)$

$3(-\sin\theta)=\frac{\sqrt{3}\sin\theta-\cos\theta}{2}$.

$-6\sin\theta=\sqrt{3}\sin\theta-\cos\theta$.

$\cos\theta=(\sqrt{3}+6)\sin\theta$.

$(\sqrt{3}+6)\frac{\sin\theta}{\cos\theta}=1$.

$\tan\theta=\frac{1}{\sqrt{3}+6}$ (shown).

13 (i) $y=(2-x)(x-5)$

$=-x^2+7x-10$.

$\frac{dy}{dx}=-2x+7$.

At A, let $\frac{dy}{dx}=2$.

$-2x+7=2$.

$x=\frac{5}{2}$.

$y=-\left(\frac{5}{2}\right)^2+7\left(\frac{5}{2}\right)-10$.

$=\frac{5}{4}$.

At C, let $\frac{dy}{dx}=-2$.

$-2x+7=-2$.

$x=\frac{9}{2}$.

$y=-\left(\frac{9}{2}\right)^2+7\left(\frac{9}{2}\right)-10$

$=\frac{5}{4}$.

A is $\left(\frac{5}{2},\frac{5}{4}\right)$ and C is $\left(\frac{9}{2},\frac{5}{4}\right)$.

13 (ii) Equation of l_1 is

$y-\frac{5}{4}=2\left(x-\frac{5}{2}\right)$.

$y=2x-\frac{15}{4}$.

At B, let $y=3\frac{1}{4}$.

$2x-\frac{15}{4}=3\frac{1}{4}$.

$x=\frac{7}{2}$.

Area of shaded region ABC

$=\left[\frac{1}{2}\left(\frac{5}{4}+3\frac{1}{4}\right)\left(\frac{7}{2}-\frac{5}{2}\right)-\int_{\frac{5}{2}}^{\frac{7}{2}}-x^2+7x-10\,dx\right]\times 2$

$=\left[2\frac{1}{4}-\left[-\frac{x^3}{3}+\frac{7}{2}x^2-10x\right]_{\frac{5}{2}}^{\frac{7}{2}}\right]\times 2$

$=\frac{2}{3}$ unit^2.