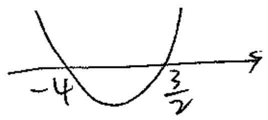
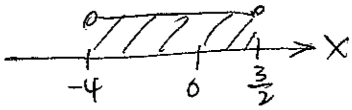
2016 N(A) AM P2 7 Oct 2016

1. $2x^2 + 5x - 12 < 0$.

$(2x-3)(x+4) < 0$.





$\therefore -4 < x < \frac{3}{2}$.

2. $xy = 2 + y^2$. ①

$2y + 1 - x = 0$, ②

②: $x = 2y + 1$ ③

Subst. ③ into ①: $(2y+1)y = 2 + y^2$.

$2y^2 + y - 2 - y^2 = 0$.

$y^2 + y - 2 = 0$.

$(y-1)(y+2) = 0$.

Subst. $y = 1$ or $y = -2$ into ③

$x = 2(1) + 1$ or $x = 2(-2) + 1$

$= 3$ $\quad = -3$.

$\therefore$ coordinates of points of intersection are $(3, 1)$ and $(-3, -2)$.

3. (i) $\therefore \frac{d}{dx}\left(\sqrt{2x^2+3}\right)$

$= \frac{1}{2}\left(2x^2+3\right)^{-\frac{1}{2}}(4x)$

$= \frac{2x}{\sqrt{2x^2+3}}$

3. (ii) $\therefore \int \frac{x}{\sqrt{2x^2+3}}\,dx$

$= \frac{1}{2}\int \frac{2x}{\sqrt{2x^2+3}}\,dx$

$= \frac{1}{2}\sqrt{2x^2+3} + c$.

$\therefore \int_0^1 \frac{x}{\sqrt{2x^2+3}}\,dx$

$= \left[\frac{1}{2}\sqrt{2x^2+3}\right]_0^1$

$= \frac{1}{2}\sqrt{5} - \frac{1}{2}\sqrt{3}$.

$= \frac{\sqrt{5}-\sqrt{3}}{2}$.

4. $y - 5x - 4 = 0$. ①

$y = 2x^2 + kx + 12$ ②

①: $y = 5x + 4$ ③

Subst. ③ into ②: $5x + 4 = 2x^2 + kx + 12$.

$2x^2 + (k-5)x + 8 = 0$. ④

Since ① is a tangent to ②, discriminant of ④ $= 0$.

$(k-5)^2 - 4(2)(8) = 0$.

$(k-5)^2 - 64 = 0$.

$(k-5+8)(k-5-8) = 0$.

$(k+3)(k-13) = 0$.

$\therefore k = -3$ or $k = 13$.

5. (i) $y = \frac{2-x^2}{x^2+1}$

$\therefore \frac{dy}{dx} = \frac{(x^2+1)(-2x) - (2-x^2)(2x)}{(x^2+1)^2}$

$= \frac{2x(-x^2-1-2+x^2)}{(x^2+1)^2}$

$= \frac{-6x}{(x^2+1)^2}$ where $a = -6$.

5. (ii) Gradient of tangent is 0 at $x=0$ / Horizontal tangent / Tangent at $x=0$ is parallel to x axis / $\perp$ y axis.

5. (iii) For y to be decreasing, $\frac{dy}{dx} < 0$.

$$\frac{-6x}{(x^2+1)^2} < 0.$$

Since $(x^2+1)^2 > 0$, $-6x < 0$.

$\therefore x > 0$.

6. (i) $\therefore (\sqrt{5}-\sqrt{2})^3$

$$= (\sqrt{5})^3 + \binom{3}{1}(\sqrt{5})^2(-\sqrt{2}) + \binom{3}{2}(\sqrt{5})(-\sqrt{2})^2 + (-\sqrt{2})^3$$

$$= 5\sqrt{5} - 15\sqrt{2} + 6\sqrt{5} - 2\sqrt{2}.$$

$$= 11\sqrt{5} - 17\sqrt{2} \quad \text{where } a = 11,\ b = 17.$$

6. (ii) $\therefore p^3 - q^3 = (p-q)(p^2+pq+q^2)$

$$\frac{5^{\frac{3}{2}} - 2^{\frac{3}{2}}}{5^{\frac{1}{2}} - 2^{\frac{1}{2}}}$$

$$= \frac{(5^{\frac{1}{2}} - 2^{\frac{1}{2}})(5 + 5^{\frac{1}{2}}.2^{\frac{1}{2}} + 2)}{5^{\frac{1}{2}} - 2^{\frac{1}{2}}}$$

$$= 5 + 10^{\frac{1}{2}} + 2$$

$$= 7 + \sqrt{10}. \quad \text{where } c = 7,\ d = 1.$$

7. (i) Gradient of AC $= \frac{4-(-6)}{1-6}$

$= -2.$

$\therefore$ Equation of AC is

$$y - 4 = -2(x-1)$$

$$y + 2x = 6.$$

7 (ii) Gradient of $AB = \dfrac{4-(-2)}{1-7}$

$= -1.$

Gradient of $DX = 1$ $(DX \perp AB)$

Equation of DX is

$y-(-7) = (1)[x-(-2.5)]$

$DX: \; y = x - 4.5$ ①

$AC: \; y + 2x = 6$ ②

Subst. ① into ②: $x - 4.5 + 2x = 6.$

$3x = 10.5$

Subst. $x = 3.5$ into ①

$y = 3.5 - 4.5$

$= -1$

$\therefore$ coordinates of P is $(3\tfrac{1}{2}, -1)$.

7 (iii) $\therefore$ midpoint of AC is

$\left(\dfrac{1+6}{2}, \dfrac{4+(-6)}{2}\right)$ or $(3\tfrac{1}{2}, -1)$, which is P.

7 (iv) $\therefore$ Area of $\triangle ADC$

$= \dfrac{1}{2}\begin{vmatrix} 1 & -2.5 & 6 & 1 \\ 4 & -7 & -6 & 4 \end{vmatrix}$

$= \frac{1}{2}(32) - \frac{1}{2}(-58)$

$= 45$ units^2.

Since P is midpoint of AC and $AP = PC$,

$\therefore$ area of $\triangle PDC = \dfrac{1}{2} \times$ area of $\triangle ADC$

$= \frac{1}{2} \times 45$ units^2

$= 22\tfrac{1}{2}$ units^2

8 (a) $\therefore$ LHS $= \operatorname{cosec}\theta + \tan\theta \sec\theta$

$= \operatorname{cosec}\theta + \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\right)$

$= \operatorname{cosec}\theta + \frac{\sin\theta}{\cos^2\theta} \times \frac{\sin\theta}{\sin\theta}$

$= \operatorname{cosec}\theta + \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\sin\theta}\right)$

$= \operatorname{cosec}\theta + \tan^2\theta \operatorname{cosec}\theta$

$= \operatorname{cosec}\theta(1 + \tan^2\theta)$

$= \operatorname{cosec}\theta \sec^2\theta.$

$=$ RHS (shown)

8. (b) (i) $\therefore \cos 2x - \sin 2x$

$= \sqrt{1^2+1^2} \cos(2x + \tan^{-1}\frac{1}{1})$

$= \sqrt{2} \cos(2x + 45^\circ)$ Where $R = \sqrt{2}$.

8. (b) (ii) $\therefore y = 6\cos^2 x - 6\cos x \sin x$

$= 6\left(\frac{1+\cos 2x}{2}\right) - 3(2\cos x \sin x)$

$= 3 + 3\cos 2x - 3\sin 2x$

$= 3 + 3(\cos 2x - \sin 2x)$

$= 3 + 3\sqrt{2}\cos(2x + 45^\circ)$ (shown)

8. (b)(iv) For $y=0$,

$3 + 3\sqrt{2}\cos(2x+45^\circ) = 0$.

$\cos(2x+45^\circ) = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}}$

basic $\angle$ $(2x+45^\circ) = 45^\circ$.

Since $x > 0$, $2x + 45^\circ > 45^\circ$

$2x + 45^\circ = 180^\circ - 45^\circ$ (2nd quadrant).

$2x + 45^\circ = 135^\circ$.

$\therefore x = 45$.

9. (i) $k\frac{d^2y}{dx^2} = l - x$

$$\frac{d^2y}{dx^2} = \frac{l}{k} - \frac{1}{k}x.$$

$$\frac{dy}{dx} = \int \frac{d^2y}{dx^2}\,dx$$

$$= \int \frac{l}{k} - \frac{1}{k}x \; dx$$

$$= \frac{l}{k}x - \frac{1}{2k}x^2 + c \quad \text{for some constant } c.$$

When $x=0$, $\frac{dy}{dx}=0$.

$$\Rightarrow \frac{l}{k}(0) - \frac{1}{2k}(0)^2 + c = 0.$$

$$\Rightarrow c = 0$$

$$\therefore \frac{dy}{dx} = \frac{l}{k}x - \frac{1}{2k}x^2.$$

9. (ii) Deflection, $y = \int \frac{dy}{dx}\,dx$

$$= \int \frac{l}{k}x - \frac{1}{2k}x^2\,dx$$

$$= \frac{l}{2k}x^2 - \frac{1}{2k(3)}x^3 + d \quad \text{for some constant } d$$

$$= \frac{l}{2k}x^2 - \frac{1}{6k}x^3 + d$$

When $x=0$, $y=0$.

$$\frac{l}{2k}(0)^2 - \frac{1}{6k}(0)^3 + d = 0. \Rightarrow d = 0.$$

$$\Rightarrow y = \frac{l}{2k}x^2 - \frac{1}{6k}x^3$$

when $x = l$, $y = \frac{l}{2k}(l)^2 - \frac{1}{6k}(l)^3$

$$= \frac{l^3}{2k} - \frac{l^3}{6k}$$

$$= \frac{l^3}{3k}$$

When $x = \frac{1}{2}l$, $y = \frac{l}{2k}\left(\frac{1}{2}l\right)^2 - \frac{1}{6k}\left(\frac{1}{2}l\right)^3$

$$= \frac{l^3}{8k} - \frac{l^3}{48k}$$

$$= \frac{5l^3}{48k}$$

$$\therefore \frac{\left(\frac{l^3}{3k}\right)}{\left(\frac{5l^3}{48k}\right)} = \frac{\left(\frac{1}{3}\right)}{\left(\frac{5}{48}\right)} = \frac{16}{5} \quad \text{(shown)}.$$

9. (iii) When $x = \frac{1}{2}l$, at mid point of the beam,

$$\frac{dy}{dx} = \frac{l}{K}\left(\frac{1}{2}l\right) - \frac{1}{2k}\left(\frac{1}{2}l\right)^2$$

$$= \frac{l^2}{2K} - \frac{l^2}{8k}$$

$$= \frac{3l^2}{8k}.$$

gradient of tangent at $x = \frac{1}{2}l$ is $-\tan 1.79° \approx -0.03125$.

$$\Rightarrow \quad \frac{3l^2}{8K} = -0.03125 \text{ (4sf.)}$$

$$\frac{3}{8}\left(\frac{l^2}{K}\right) = -0.03125$$

$$\frac{l^2}{K} = -0.03125 \div \frac{3}{8}.$$

$$\therefore \quad \frac{K}{l^2} = -12.$$