1\. (i) $P = P_0 e^{kt}$

$\ln P = \ln P_0 + \ln e^{kt}$

$\ln P = \ln P_0 + kt$

plot $\ln P$ against t : Gradient of graph $= k$, $(\ln P)$ intercept $= \ln P_0$.

t	0	5	10	15
P	2.00	2.44	3.00	3.65
$\ln P$ (2d.p)	0.69	0.89	1.10	1.29

(ii) From graph, gradient of straight line $= \frac{0.43}{10.6}$

$= 0.0406$ (3sf.)

$\Rightarrow k = 0.0406$

$\ln P$ - intercept $= 0.69$

$\Rightarrow \ln P_0 = 0.69$ (2d.p.)

$P_0 = 2.$

$\therefore P_0 = 2$, $k = 0.0406$ (3sf.)

(iii) From graph, when $t = 20$, $\ln P = 1.50$

$\Rightarrow P = e^{1.50}$

$= 4.48$ (nearest cent)

$\therefore$ price of a company share on 1st Jan 2015 $= \$4.48$.

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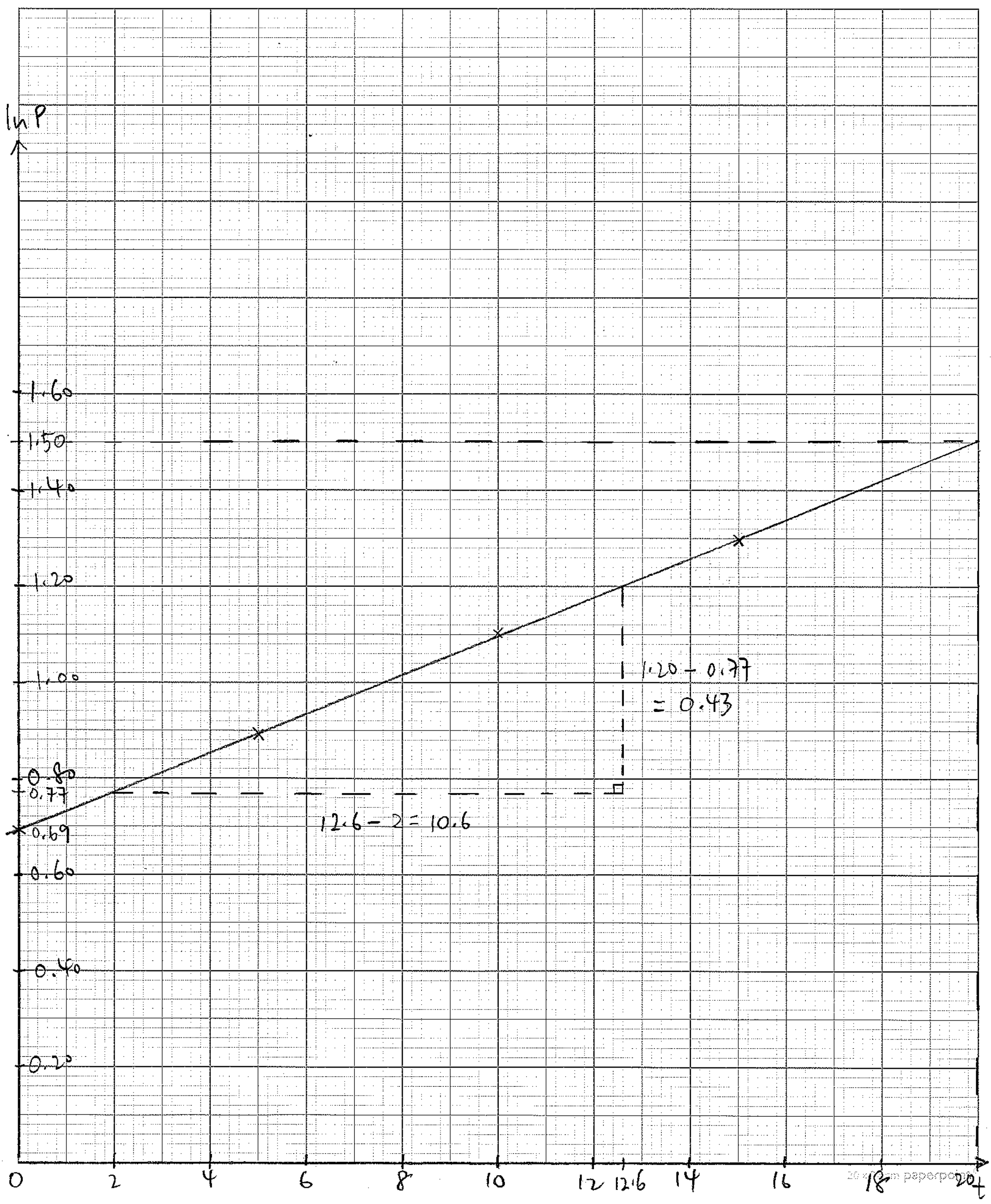


2. (i) $(1-2x)^2(1-px)^6$

$= (1-4x+4x^2)(1-6px+15p^2x^2-\dots)$

Coefficient of $x^2 = (1)(15p^2)+(-4)(-6p)+(4)(1)$

$= 15p^2+24p+4.$

$\Rightarrow 15p^2+24p+4=16.$

$15p^2+24p-12=0.$

$5p^2+8p-4=0.$

$(5p-2)(p+2)=0.$

$\therefore p=\frac{2}{5}$ or -2

(ii) General term of $(1-px)^6 = \binom{6}{r}(1)^{6-r}(-px)^r$

$= \binom{6}{r}(-p)^r(x)^r$

For x^3 term, let $r=3$.

When $p=\frac{2}{5}$, $\therefore$ coefficient of $x^3 = \binom{6}{3}\left(-\frac{2}{5}\right)^3$

$= -\frac{32}{25}.$

When $p=-2$, $\therefore$ coefficient of $x^3 = \binom{6}{3}(2)^3$

$= 160.$

3. (i) $\therefore \cos 3x$

$= \cos(2x+x)$

$= \cos 2x\cos x - \sin 2x\sin x$

$= (1-2\sin^2 x)\cos x - 2\sin^2 x\cos x$

$= \cos x(1-2\sin^2 x-2\sin^2 x)$

$= \cos x(1-4\sin^2 x)$ (shown)

(ii) $2\cos 3x = 15\sin x\cos x$

$2\cos x(1-4\sin^2 x) - 15\sin x\cos x = 0.$

$\cos x(2-8\sin^2 x-15\sin x)=0.$

$-\cos x(8\sin^2 x+15\sin x-2)=0.$

$\cos x(8\sin x-1)(\sin x+2)=0.$

$\cos x=0$, $\sin x=\frac{1}{8}$ or $\sin x=-2$ (N.A. $\because -1\le\sin x\le 1$).

$x=90^\circ, 270^\circ$ $x=\sin^{-1}\frac{1}{8}, 180^\circ-\sin^{-1}\frac{1}{8}$

$= 7.18^\circ, 172.82^\circ$

$\therefore x = 7.2^\circ, 90^\circ, 172.8^\circ, 270^\circ.$

4. (i) $x^2+2x+5=0.$

$\alpha+\beta=-2$

$\alpha\beta=5.$

$\therefore \alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$

$=(-2)[(-2)^2-3(5)]$

$=22.$

(ii) $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$

$=\frac{22}{(5)^2}$

$=\frac{22}{25}$

$\left(\frac{\alpha}{\beta^2}\right)\left(\frac{\beta}{\alpha^2}\right)=\frac{\alpha\beta}{\alpha^2\beta^2}$

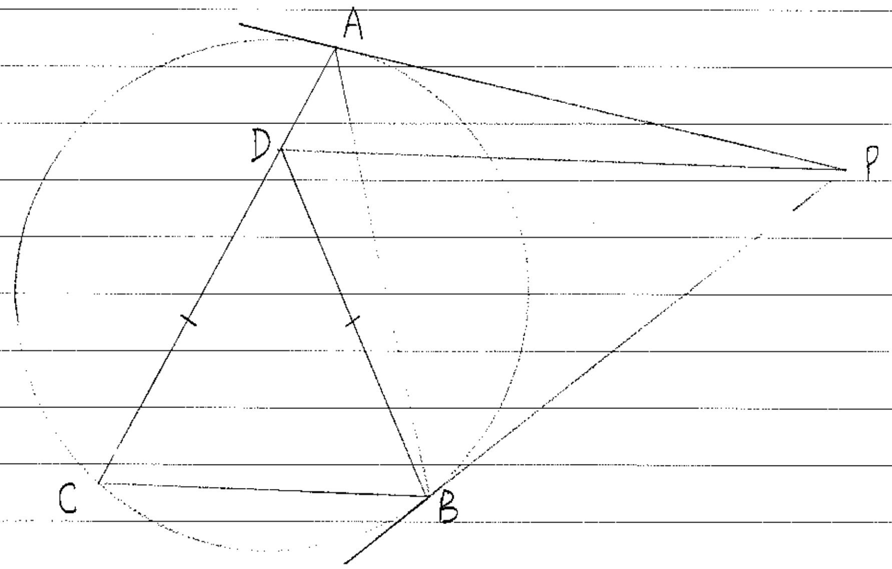
$=\frac{1}{\alpha\beta}$

$=\frac{1}{5}.$

$\therefore$ Equation with roots $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$ is

$x^2-\frac{22}{25}x+\frac{1}{5}=0$ or $25x^2-22x+5=0.$

5. (i)



Let $\angle BCD$ be θ.

$\angle BCD=\angle CBD=\theta$ ($DC=DB$)

$\angle PAB=\angle BCD=\theta$ (Alternate segment Theorem)

$\angle PAB=\angle ABP=\theta$ ($PA=PB$, tangents from ext. point are equal.)

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5. (i) (continued)

$\angle ADB = \angle BCD + \angle CBD$ (ext $\angle$ of ΔBCD).

$= 2\theta$

$\angle APB = 180° - \angle PAB - \angle ABP$ ($\angle$ sum of Δ)

$= 180° - \theta - \theta$

$= 180° - 2\theta$.

$\therefore \angle APB + \angle ADB = 180° - 2\theta + 2\theta$

$= 180°$ (shown)

(ii) Since A, P, B, D lie on a circle, $\angle BDP = \angle PAB$ ($\angle$s in same segment.)

$= \theta$.

$\angle CBD = \theta = \angle BDP$

$\therefore PD // BC$ ($\angle CBD$ and $\angle BDP$ are alternate $\angle$s)

6. (i) $y = (x-2)(2x-5)^3$

$\therefore \frac{dy}{dx} = 3(x-2)(2x-5)^2(2) + (2x-5)^3$

$= (2x-5)^2(6x-12+2x-5)$

$= (2x-5)^2(8x-17)$ where $a = 8$, $b = -17$.

(ii) For decreasing y, $\frac{dy}{dx} < 0$.

$(2x-5)^2(8x-17) < 0$.

Since $(2x-5)^2 > 0$ for all x where $x \neq \frac{5}{2}$, $8x-17 < 0$.

$\therefore x < 2\frac{1}{8}$.

(iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

When $x=3$, $\frac{dy}{dx} = 7$, $y = 1$.

$\Rightarrow 0.35 = 7 \times \frac{dx}{dt}$

$\frac{dx}{dt} = 0.05$

$\therefore$ rate of change of x is 0.05 units/s.

(iv) $z = y^2$

$\frac{dz}{dy} = 2y$.

$\frac{dz}{dt} = \frac{dz}{dy} \times \frac{dy}{dt}$

When $x=3$, $y=1$, $\frac{dz}{dy} = 2$. $\Rightarrow \therefore \frac{dz}{dt} = 2\frac{dy}{dt}$ (shown).

7. (i) $u = 2^x$

$2^{2x-1} = 2^{x+2} - 6.$

$(2^x)^2 \cdot (\frac{1}{2}) = (2^x) \cdot (2^2) - 6.$

$\frac{1}{2}u^2 = 4u - 6 \Rightarrow \therefore$ Equation in u is $u^2 - 8u + 12 = 0$.

(ii) $(u-6)(u-2) = 0.$

$u = 2$ or $u = 6$

$\Rightarrow 2^x = 2$ or $2^x = 6.$

$x = 1$ or $x = \log_2 6$

$= 2.6$ (1 d.p.)

$\therefore x = 1$ or 2.6

(iii) From (i), using $2^x = u$,

$2^{2x-1} = 2^{x+2} - k$ simplifies to $\frac{1}{2}u^2 = 4u - k.$

$\Rightarrow u^2 - 8u + 2k = 0.$

Solving u, discriminant of equation $= (-8)^2 - 4(1)(2k)$

$= 64 - 8k$

If $k > 8$, $-8k < -64$

$64 - 4k < 0$

$\Rightarrow$ Discriminant < 0 (no real roots)

$\therefore 2^{2x-1} = 2^{x+2} - k$ has no solution if $k > 8$.

8. (i) $f(x) = x^3 - 3x^2 + 4x - 12.$

Let $x = 3$, $f(3) = (3)^3 - 3(3)^2 + 4(3) - 12$

$= 0.$

$\Rightarrow x - 3$ is a factor of $f(x)$.

Let $f(x) = x^3 - 3x^2 + 4x - 12$

$= (x-3)(x^2 + ax + 4)$ for some constant a.

Comparing coefficients of x, $4 = 4 + (-3)(a) \Rightarrow a = 0.$

$\therefore f(x) = x^3 - 3x^2 + 4x - 12$

$= (x-3)(x^2+4).$

8. (ii) When $f(x)=0$, $(x-3)(x^2+4)=0$.

$x-3=0$ or $x^2+4=0$.

$\Rightarrow x=3$

Since $x^2+4 \geqslant 4 > 0$, there is no solution to $x^2+4=0$.

$\therefore$ $x=3$ is the only real root to $f(x)=0$.

(iii) $y = f(x)+kx$

$\frac{dy}{dx} = f'(x)+k$

$= 3x^2-6x+4+k$

$\frac{d^2y}{dx^2} = 6x-6$

Given $\frac{d^2y}{dx^2}=0$, $6x-6=0$. $\Rightarrow x=1$.

At stationary points, $\frac{dy}{dx}=0$

Since $x=1$ is the solution to $\frac{d^2y}{dx^2}=0$,

$x=1$ is the stationary value to $\frac{dy}{dx}=0$.

$\Rightarrow$ $3(1)^2-6(1)+4+k=0$.

$\therefore k=-1$.

9. (i) $y = x^3+2x^2-3x$.

$\frac{dy}{dx} = 3x^2+4x-3$.

At A, $x=\frac{2}{3}$, $\frac{dy}{dx} = 3(\frac{2}{3})^2+4(\frac{2}{3})-3$

$= 1$

$\therefore$ Gradient of curve at $A = 1$.

(ii) At B, gradient of tangent to curve $= 1$.

$\Rightarrow$ $3x^2+4x-3=1$.

$3x^2+4x-4=0$.

$(3x-2)(x+2)=0$.

$x=\frac{2}{3}$ or $x=-2$.

$\Rightarrow \therefore$ x-coordinate of B is -2.

(iii) $\therefore$ Area of shaded region $= \int_{-2}^{0} y\,dx + \left|\int_{0}^{\frac{2}{3}} y\,dx\right|$

$= \left[\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{-2}^{0}$

$+ \left|\left[\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{0}^{\frac{2}{3}}\right|$

$= 7\frac{61}{81}$ units^2.

10. (i) When $t=0$, $v = 30e^0 + 20$
$= 50$.
$\therefore p = 50$.

(ii) At B, $v = 80$
$30e^{25t} + 20 = 80$.
$e^{25t} = 2$.
$25t = \ln 2$.
$t = \frac{1}{25}\ln 2$
$\therefore$ Time taken in seconds $= \frac{1}{25}\ln 2 \times 60 \times 60$
≈ 100

(iii) $e^{25t} > 0$.
$30e^{25t} + 20 > 20 > 0$
Since $v > 0$,
$\therefore$ distance between A and B $= \int_0^{\frac{1}{25}\ln 2} v \, dt$
$= \left[\frac{30}{25}e^{25t} + 20t \right]_0^{\frac{1}{25}\ln 2}$
$= \frac{6}{5}e^{\ln 2} + 20\left(\frac{1}{25}\ln 2\right) - \frac{6}{5}e^0$.
$= \frac{4}{5}\ln 2 + \frac{6}{5}$
$= \frac{2}{5}(2\ln 2 + 3)$ or 1.75 km (3 s.f.)

(iv) $a = \frac{dv}{dt}$
$= 25(30e^{25t})$
$= 750e^{25t}$
$\therefore$ Acceleration of car between A and B $= 750e^{25t}$.

11. (i) C_1: $x^2 + y^2 - 4x - 2y = 95$.
$x^2 - 4x + y^2 - 2y = 95$.
$x^2 - 4x + (-2)^2 + y^2 - 2y + (-1)^2 = 95 + (-2)^2 + (-1)^2$
$(x-2)^2 + (y-1)^2 = 100$.
$\therefore$ Coordinates of centre A is $(2, 1)$.
$\therefore$ Radius of $C_1 = \sqrt{100} = 10$ units.

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11. (ii) Subst. $x = 10$, $y = 7$ into C_1

$LHS = 10^2 + 7^2 - 4(10) - 2(7)$

$= 95 = RHS$

$\therefore P(10, 7)$ lies on C_1.

(iii) Gradient of radius $AP = \frac{7-1}{10-2}$

$= \frac{3}{4}$.

Gradient of tangent to C_1 at $P = -\frac{4}{3}$ (tangent $\perp$ radius at P).

$\Rightarrow y - 7 = -\frac{4}{3}(x-10)$

$y = -\frac{4}{3}x + \frac{61}{3}$

$3y + 4x = 61$

$\therefore$ Equation of tangent to C_1 at P is $3y + 4x = 61$.

(iv) Midpoint of AP is $(6,4) \Rightarrow$ centre of C_2 is $(6,4)$.

Length of radius AP of $C_1 = 10$ units

Radius of $C_2 = \frac{10}{2} = 5$ units.

$\therefore$ Equation of C_2 is

$(x-6)^2 + (y-4)^2 = 5^2$

$(x-6)^2 + (y-4)^2 = 25$.

(v) Since AP is diameter of C_2 and $AP \perp$ tangent at P,

$\therefore$ Equation of tangent to C_2 at P is $3y + 4x = 61$.