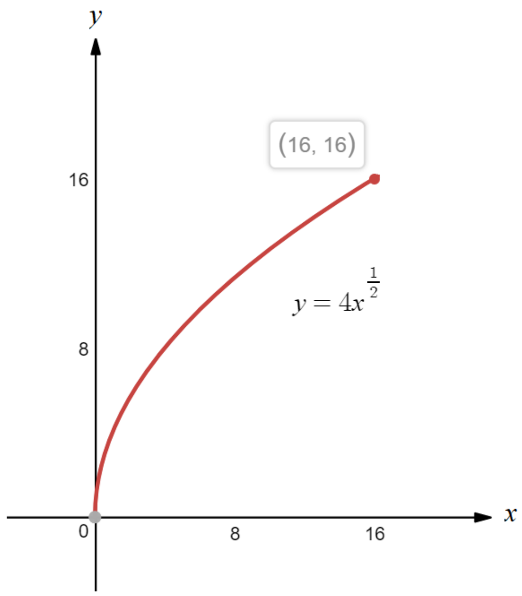
Suggested Solutions of 2017 Nov O Level Additional Mathematics Paper 1 Syllabus 4047

1	$\frac{d^2y}{dx^2}=8-6x$ $\frac{dy}{dx}=\int 8-6x dx$ $=8x-3x^2+c$, for some constant c At P, gradient = 3. $\Rightarrow 8(2)-3(2)^2+c=3$ $c=-1$ $y=\int \frac{dy}{dx}dx$ $=\int 8x-3x^2-1 dx$ $= 4x^2 - x^3 - x + d$, for some constant d At P, $x = 2$ and $y = 8$. $\Rightarrow 4(2)^2-(2)^3-(2)+d=8$ $d = 2$ $\therefore$ Equation of curve is $y=-x^3+4x^2-x+2$	[1] [1] [1] [1] [1] [1]
2(i)		[1] domain, pass through origin [1] shape, equation labelled
2(ii)	$y=4x^{\frac{1}{2}}$ -(1) $4y = 7x + 4$ -(2) Substitute -(1) into -(2), $16x^{\frac{1}{2}}=7x+4$ $7\left(x^{\frac{1}{2}}\right)^2-16\left(x^{\frac{1}{2}}\right)+4=0$ $\left(x^{\frac{1}{2}}-2\right)\left(7x^{\frac{1}{2}}-2\right)=0$ $x^{\frac{1}{2}}=2$ or $x^{\frac{1}{2}}=\frac{2}{7}$	[1] [1]

	$x = 2^2 = 4$ or $x = \left(\frac{2}{7}\right)^2 = \frac{4}{49}$ When $x = 4$, $y = 4(4)^{\frac{1}{2}} = 8$ When $x = \frac{4}{49}$, $y = 4\left(\frac{4}{49}\right)^{\frac{1}{2}} = 1\frac{1}{7}$ $\therefore$ Coordinates of the points of intersection are $(4,8)$ and $\left(\frac{4}{49}, 1\frac{1}{7}\right)$.	[1] [1]
3	When $x = 0.04$, $\frac{1}{\sqrt{x}} = 5$. When $y = 0.25$, $\frac{1}{y} = 4$. When $x = 1.00$, $\frac{1}{\sqrt{x}} = 1$. When $y = 0.50$, $\frac{1}{y} = 2$. Let A be the point $(5,4)$ and B be the point $(1,2)$. When $\frac{1}{y}$ is plotted against $\frac{1}{\sqrt{x}}$, gradient of straight line $=$ gradient of AB $= \frac{4-2}{5-1} = \frac{1}{2}$ Equation between variables x and y is $\frac{1}{y} - 2 = \frac{1}{2}\left(\frac{1}{\sqrt{x}} - 1\right)$ $\frac{1}{y} = \frac{1}{2\sqrt{x}} + \frac{3}{2}$ When $x = 9$, $\frac{1}{y} = \frac{1}{2\sqrt{9}} + \frac{3}{2} = \frac{5}{3}$ $\therefore y = \frac{3}{5}$	[1] [1] [1] [1] [1]
4	$7x^2 - 3x + 1 = 0$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{7}$ -(1) $\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{7}$ -(2) -(2): $\alpha\beta = 7$ -(1): $\frac{\beta + \alpha}{\alpha\beta} = \frac{3}{7}$ Substitute $\alpha\beta = 7$, $\frac{\beta + \alpha}{7} = \frac{3}{7}$. $\Rightarrow \alpha + \beta = 3$	[1] [1] [1]

	$\alpha^2+\beta^2$ $=(\alpha+\beta)^2-2\alpha\beta$ $=(3)^2-2(7)$ $=-5$	[1]
	$\alpha^2\beta^2$ $=(\alpha\beta)^2$ $=(7)^2$ $=49$	[1]
	$\therefore$ Quadratic equation with roots α^2 and β^2 is $x^2+5x+49=0$.	[1]
5(i)	$\therefore \dfrac{\sec x+\operatorname{cosec} x}{\sec x-\operatorname{cosec} x}$ $=\dfrac{\left(\dfrac{1}{\cos x}+\dfrac{1}{\sin x}\right)}{\left(\dfrac{1}{\cos x}-\dfrac{1}{\sin x}\right)}$ $=\dfrac{\left(\dfrac{\sin x+\cos x}{\sin x\cos x}\right)}{\left(\dfrac{\sin x-\cos x}{\sin x\cos x}\right)}$	[1]
	$=\dfrac{\sin x+\cos x}{\sin x-\cos x}$ $=\dfrac{\left(\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\cos x}\right)}{\left(\dfrac{\sin x}{\cos x}-\dfrac{\cos x}{\cos x}\right)}$	[1]
	$=\dfrac{\tan x+1}{\tan x-1}$ (shown)	[1]
5(ii)	$\dfrac{\sec x+\operatorname{cosec} x}{\sec x-\operatorname{cosec} x}=\dfrac{5}{2}$ $\dfrac{\tan x+1}{\tan x-1}=\dfrac{5}{2}$ $2\tan x+2=5\tan x-5$ $3\tan x=7$	[1]
	$\tan x=\dfrac{7}{3}$ basic $\angle x=\tan^{-1}\left(\dfrac{7}{3}\right)=1.166$ (4 s.f.)	[1]
	$x=1.166, \pi+1.166$ $\therefore x=1.17, 4.31$ (3 s.f.)	[1]
6(i)	Total length of wire netting used is 288 m. $\Rightarrow 6x+4y=288$ $3x+2y=144$	[1]

	$y=72-\frac{3}{2}x$ $\therefore$ Total area of the three tennis courts is $A=3xy$ $=3x\left(72-\frac{3}{2}x\right)$ $=216x-\frac{9}{2}x^2$ (shown)	[1] [1]
6(ii)	$\frac{dA}{dx}=216-9x$ At stationary points, $\frac{dA}{dx}=0$. $216-9x=0 \Rightarrow x=24$ $y=72-\frac{3}{2}(24)=36$ $\therefore$ Dimensions of each tennis court are 24 m by 36 m.	[1] [1] [1]
7(i)	Area of triangle $ABC=\frac{1}{4}\left(9+\sqrt{3}\right)$ $\Rightarrow \frac{1}{2}\times AB\times AC\times \sin\angle BAC=\frac{1}{4}\left(9+\sqrt{3}\right)$ $\frac{1}{2}\times\left(\sqrt{3}+1\right)\times AC\times\sin 60^\circ=\frac{9+\sqrt{3}}{4}$ $\frac{\sqrt{3}+1}{2}\times AC\times\frac{\sqrt{3}}{2}=\frac{9+\sqrt{3}}{4}$ $AC\times\left(3+\sqrt{3}\right)=9+\sqrt{3}$ $AC=\frac{9+\sqrt{3}}{3+\sqrt{3}}\times\frac{3-\sqrt{3}}{3-\sqrt{3}}$ $=\frac{27-9\sqrt{3}+3\sqrt{3}-3}{(3)^2-\left(\sqrt{3}\right)^2}$ $=\frac{24-6\sqrt{3}}{6}=4-\sqrt{3}$, where $a=4, b=-1$	 [1] [1] [1]
7(ii)	Using Cosine Rule, $\therefore BC^2=AB^2+AC^2-2\times AB\times AC\times\cos 60^\circ$ $=\left(\sqrt{3}+1\right)^2+\left(4-\sqrt{3}\right)^2-2\left(\sqrt{3}+1\right)\left(4-\sqrt{3}\right)\left(\frac{1}{2}\right)$ $=3+2\sqrt{3}+1+16-8\sqrt{3}+3-\left(4\sqrt{3}-3+4-\sqrt{3}\right)$ $=22-9\sqrt{3}$, where $c=22, d=-9$	 [1] [1] [1]
8(i)	x^2+9 $3x-1\,\overline{)\,3x^3-x^2+27x-9}$ $-(3x^3-x^2)$ $27x-9$ $-(27x-9)$ 0 $\therefore$ Quotient is x^2+9 and the remainder is 0.	 [1]

8(ii)	$\frac{6+11x-5x^2}{3x^3-x^2+27x-9}=\frac{6+11x-5x^2}{(3x-1)(x^2+9)}$ Let $\frac{6+11x-5x^2}{(3x-1)(x^2+9)}=\frac{A}{3x-1}+\frac{Bx+C}{x^2+9}$ for some constants A, B and C. $6 + 11x - 5x^2 = A(x^2 + 9) + (Bx + C)(3x - 1)$ -(1) When $x=\frac{1}{3}$, $6+11\left(\frac{1}{3}\right)-5\left(\frac{1}{3}\right)^2=A\left[\left(\frac{1}{3}\right)^2+9\right]\Rightarrow A=1$ -(1): $6 + 11x - 5x^2 = x^2 + 9 + 3Bx^2 - Bx + 3Cx - C$ Comparing coefficients of x^2, $3B+1=-5\Rightarrow B=-2$ Comparing constant terms, $9 - C = 6 \Rightarrow C=3$ $\therefore \frac{6+11x-5x^2}{3x^3-x^2+27x-9}=\frac{1}{3x-1}+\frac{3-2x}{x^2+9}$	[1] [1] [1] [1] [1]
9(i)	$v=\int\frac{dv}{dt}dt$ $= 10t + c$, for some constant c When $t = 0$, $v = 0 \Rightarrow c=0$. $v = 10t$ When $t = 4$, $v = 10(4) = 40$. $\therefore$ Velocity at X = 40 m/s	 [1] [1]
9(ii)	$s=\int vdt$ $= 5t^2 + d$, for some constant d When $t = 0$, $s = 0 \Rightarrow c=0$. $s = 5t^2$ When $t = 4$, $s = 5(4)^2 = 80$ $\therefore$ Distance OX = 80 m	 [1] [1]
9(iii)	Velocity after T seconds, V $=\int\frac{dV}{dT}dT$ $=\int 10-kTdT$ $=10T-\frac{k}{2}T^2+h$, for some constant h and k At point X, when $T = 0$, $V = 40$. $10(0)-\frac{k}{2}(0)^2+h=40\Rightarrow h=40$. $V=10T-\frac{k}{2}T^2+40$ At point Y, when $T = 3$, $V = 0$. $\Rightarrow 10(3)-\frac{k}{2}(3)^2+40=0$ $\frac{9k}{2}=70$ $\therefore k=\frac{140}{9}$ (shown)	 [1] [1] [1]
10(i)	$\angle PBA=\angle ACB$ (Alternate Segment Theorem)	[1]

	$\angle ACB = \angle DAC$ (*BC* is parallel to *AD*) $\therefore \angle PBA = \angle DAC$ (shown)	[1] [1]
10(ii)	$\angle ACB = \angle ADB$ ($\angle$s in the same segment) Since *OB* = *OD* (radii of circle), $\angle OBD = \angle ODB$ (base $\angle$s of isosceles triangle) $= \angle ACB$ $= \angle PBA$ $\angle DAC = \angle DBC$ ($\angle$s in the same segment) Since $\angle PBA = \angle DAC$, $\angle PBA = \angle DBC = \angle OBD$. $\angle OBT = 90°$ (tangent $\perp$ radius) $\therefore \angle CBT = \angle OBT - \angle OBD - \angle DBC$ $= 90° - 2\times(\angle PBA)$ (shown)	[1] [1] [1] [1] [1]
11(i)	$y = \frac{2x+1}{x-1}$ $\frac{dy}{dx} = \frac{(x-1)(2)-(2x+1)(1)}{(x-1)^2}$ $= -\frac{3}{(x-1)^2}$ Since $x \neq 1$, $(x-1)^2 > 0$, $-\frac{3}{(x-1)^2} < 0$. Since $\frac{dy}{dx}$ can never be zero, the curve has no turning points.	 [1] [1] [1]
11(ii)	$\frac{2x+1}{x-1} = \frac{2(x-1)+3}{x-1}$ $= 2 + \frac{3}{x-1}$ $\therefore$ Area of the shaded region $=$ Area of trapezium – Area under curve $= \frac{1}{2}\times(3+5)\times(2) - \int_2^4 \frac{2x+1}{x-1}dx$ $= 8 - \int_2^4 2 + \frac{3}{x-1}dx$ $= 8 - [2x + 3\ln(x-1)]_2^4$ $= 8 - [8 + 3\ln 3 - (4 + 3\ln 1)]$ $= 4 - 3\ln 3$ or 0.704 units^2 (3 s.f.)	 [1] [1] [1] [2]
12(i)	$x^2 + y^2 + 8x - 24y + 96 = 0$ $x^2 + 8x + (4)^2 + y^2 - 24y + (12)^2 = (4)^2 + (12)^2 - 96$ $(x+4)^2 + (y-12)^2 = 64$ Centre of circle is $(-4, 12)$. Normal at *R* passes through centre $(-4, 12)$ $\because$ tangent $\perp$ radius. $\Rightarrow$ 3(12) + 4(–4) = *k* $\therefore$ *k* = 20	 [2] [1] [1] [1]
12(ii)	3*y* + 4*x* = 20 $y = \frac{20-4x}{3}$	 [1]

	When $y = 0$, $x = 5$. $\Rightarrow$ S is $(5,0)$. Substitute $y=\dfrac{20-4x}{3}$ into equation of circle, $(x+4)^2+\left(\dfrac{20-4x}{3}-12\right)^2=64$ $(x+4)^2+\left(-\dfrac{16}{3}-\dfrac{4}{3}x\right)^2=64$ $x^2+8x+16+\dfrac{256}{9}+\dfrac{128}{9}x+\dfrac{16}{9}x^2=64$ $25x^2+200x-176=0$ $x=\dfrac{4}{5}$ or $x=-\dfrac{44}{5}$ When $x=\dfrac{4}{5}$, $y=5\dfrac{3}{5}<12$ When $x=-\dfrac{44}{5}$, $y=18\dfrac{2}{5}>12$ Since R is between S and centre $(-4,12)$, R is $\left(\dfrac{4}{5},5\dfrac{3}{5}\right)$. $\therefore$ Length of RS $=\sqrt{\left(5-\dfrac{4}{5}\right)^2+\left(0-5\dfrac{3}{5}\right)^2}$ $=\sqrt{49}$ $=7$ units	[1] [1] [1] [1]