Suggested Solutions of 2017 Nov O Level Additional Mathematics Paper 2 Syllabus 4047

1	$y = e^{-x}x^2$ -(1) $\frac{dy}{dx} = e^{-x}(2x) + (-e^{-x})x^2$ $= e^{-x}(2x - x^2)$ -(2) $\frac{d^2y}{dx^2} = e^{-x}(2-2x) + (-e^{-x})(2x - x^2)$ $= e^{-x}(2 - 2x - 2x + x^2)$ $= e^{-x}(2 - 4x + x^2)$ -(3) Substitute -(1), -(2) and -(3) into $e^x\left(\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y\right) = k$, $e^x\left[e^{-x}(2 - 4x + x^2) + 2e^{-x}(2x - x^2) + e^{-x}x^2\right] = k$ $2 - 4x + x^2 + 4x - 2x^2 + x^2 = k$ $\therefore\ k = 2$	[1] [2] simplified expressions [3] [1]
2(i)	$\therefore\ \frac{d}{dx}(\tan x - x)$ $= \sec^2 x - 1$ $= \tan^2 x$ (shown)	[1] [1]
2(ii)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(\sec^2 x + 5\tan^2 x)dx = a\sqrt{3} + b\pi$ $[\tan x + 5(\tan x - x)]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = a\sqrt{3} + b\pi$ $[6\tan x - 5x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = a\sqrt{3} + b\pi$ $\left(6\tan\frac{\pi}{3} - \frac{5\pi}{3}\right) - \left(6\tan\frac{\pi}{6} - \frac{5\pi}{6}\right) = a\sqrt{3} + b\pi$ $6\sqrt{3} - 6\left(\frac{\sqrt{3}}{3}\right) - \frac{5\pi}{6} = a\sqrt{3} + b\pi$ $4\sqrt{3} - \frac{5\pi}{6} = a\sqrt{3} + b\pi$ Comparing terms, $\therefore\ a = 4,\ b = -\frac{5}{6}$	[2] [1] [1] [1]
3(i)	General term of $\left(px^3 + \frac{1}{x}\right)^9$ $= \binom{9}{r}(px^3)^{9-r}\left(\frac{1}{x}\right)^r$ $= \binom{9}{r}p^{9-r}x^{27-3r}(x^{-1})^r$	[1]

	$=\binom{9}{r}p^{9-r}x^{27-4r}$ The power of x in each term is $27 - 4r$, which yields an odd integer for any integer value of r since $4r$ is even. $\therefore$ There are no even powers of x in this expansion.	[1] [1]
3(ii)	Let $27 - 4r = 11 \Rightarrow r = 4$ When $r = 4$, coefficient of $x^{11} = \binom{9}{4}p^{9-4} = 126p^5$ Let $27 - 4r = 7 \Rightarrow r = 5$ When $r = 5$, coefficient of $x^7 = \binom{9}{5}p^{9-5} = 126p^4$ Since coefficient of x^{11} is twice the coefficient of x^7, $126p^5 = 2 \times 126p^4$ Since $p \neq 0$, $\therefore$ $p = 2$.	[1] [1] [1] [1]
4(i)	$y = \frac{6}{\sqrt{x}} + x = 6x^{-\frac{1}{2}} + x$ $\frac{dy}{dx} = -3x^{-\frac{3}{2}} + 1 = -\frac{3}{\sqrt{x^3}} + 1$ $\frac{d^2y}{dx^2} = \frac{9}{2}x^{-\frac{5}{2}} = \frac{9}{2\sqrt{x^5}}$ At stationary points, $\frac{dy}{dx} = 0$. $-\frac{3}{\sqrt{x^3}} + 1 = 0$ $\sqrt{x^3} = 3$ $x^3 = 3^2 = 9$ Since $x^3 > 0$, $x^5 > 0$, $\frac{d^2y}{dx^2} = \frac{9}{2\sqrt{x^5}} > 0$ (min.) and the stationary point M is a minimum. $\therefore$ x-coordinate of M satisfies the equation $x^3 = 9$. (shown)	[1] [1] [2]
4(ii)	At $A(1,7)$, $\frac{dy}{dx} = -\frac{3}{\sqrt{(1)^3}} + 1 = -2$. Equation of tangent at A is $y - 7 = -2(x-1)$ $y = -2x + 9$ -(1) At $B(4,7)$, $\frac{dy}{dx} = -\frac{3}{\sqrt{(4)^3}} + 1 = \frac{5}{8}$. Equation of tangent at B is $y - 7 = \frac{5}{8}(x-4)$ $y = \frac{5}{8}x + \frac{9}{2}$ -(2)	[1] [1]

	Substitute -(1) into -(2), $-2x+9=\frac{5}{8}x+\frac{9}{2} \Rightarrow x=1\frac{5}{7}$ x-coordinate of P is $1\frac{5}{7}$. x-coordinate of M is $\sqrt[3]{9} \approx 2.08$. Since $1\frac{5}{7} < \sqrt[3]{9}$, $\therefore$ x-coordinate of P is less than x-coordinate of M.	[2] [1]
5(i)	$\log_5(x-1)-\log_5(x+1)=1+\log_5\frac{1}{7}$ $\log_5\left(\frac{x-1}{x+1}\right)=\log_5 5+\log_5\frac{1}{7}$ $\log_5\left(\frac{x-1}{x+1}\right)=\log_5\frac{5}{7}$ $\frac{x-1}{x+1}=\frac{5}{7}$ $7x-7=5x+5$ $2x=12$ $\therefore\ x=6$	 [2] [1] [1]
5(ii)	$\log_y 100=\lg y$ $\frac{\lg 100}{\lg y}=\lg y$ $(\lg y)^2=2$ $\lg y=\sqrt{2}$ or $-\sqrt{2}$ $y=10^{\sqrt{2}}$ or $10^{-\sqrt{2}}$ $\therefore\ y=26$ (2 s.f.) or 0.039 (2 s.f.)	 [1] [2] [2]
6(i)	$y=9x^2+(2m+1)x+1+c$ -(1) $y=mx+c$ -(2) Substitute -(1) into -(2), $9x^2+(2m+1)x+1+c=mx+c$ $9x^2+(m+1)x+1=0$ -(3) Since -(2) is a tangent to -(1), -(3) has real and equal roots. Discriminant = 0 $(m+1)^2-4(9)(1)=0$ $(m+1)^2=36$ $m+1=6$ or -6 $m=5$ or -7 (N.A.) $\therefore$ The positive value of m is 5.	 [1] [1] [1] [1]
6(ii)	Since the curve passes through $(-2,19)$, $9(-2)^2+(10+1)(-2)+1+c=19 \Rightarrow c=4$ Equation of tangent at P is $y=5x+4$. Equation of curve is $y=9x^2+11x+5$. $\frac{dy}{dx}=18x+11$	 [1]

	Since the line $y = 5x + 4$ is a tangent to the curve, let $\frac{dy}{dx} = 5$. $18x + 11 = 5 \Rightarrow x = -\frac{1}{3}$ Substitute $x = -\frac{1}{3}$ into $y = 5x + 4$, $y = 2\frac{1}{3}$. $\therefore$ P is $\left(-\frac{1}{3}, 2\frac{1}{3}\right)$.	[2] [1]
6(iii)	Since L meets the curve at one point only and L is not a tangent to the curve, $\therefore$ L is the line of symmetry of the quadratic curve which is a normal to the curve at the minimum point.	[1]
7(a)(i)	$P = 100e^{-kt}$ When $t = 0$, $P = 100$. When $t = 5730$, $P = 50$. $\Rightarrow 100e^{-k(5730)} = 50$ $e^{-5730k} = \frac{1}{2}$ $-5730k = \ln\frac{1}{2}$ $\therefore k = -\frac{1}{5730}\ln\frac{1}{2} \approx 0.000\ 120\ 97 = 0.000\ 121$ (3 s.f.)	[1] [1] [1]
7(a)(ii)	When $t = 8000$, $P = 100e^{-0.00012097(8000)} = 38.0$ (3 s.f.) $\therefore$ Percentage of carbon-14 indicating fossil age of 8000 years = 38.0%	[2]
7(b)	$S = \lg\frac{I}{c}$ Let I_0 denote the intensity of an event of size 2.4 and I_1 as the intensity of an event which has 50 times the intensity of an event of size 2.4, so $I_1 = 50I_0$. When $S = 2.4$, $\lg\frac{I_0}{c} = 2.4$ $\Rightarrow \frac{I_0}{c} = 10^{2.4}$ Intensity of event size 2.4, $I_0 = (10^{2.4})c$ for some constant c $I_1 = 50I_0 = 50(10^{2.4})c$ Let S_1 denote the size of the event which has intensity I_1. $\therefore S_1 = \lg\frac{I_1}{c}$ $= \lg\frac{50(10^{2.4})c}{c}$ $= \lg 50(10^{2.4})$ $= 4.1$ (1 d.p.)	[1] [2] [1]
8(a)	$y = \ln(3x-1)$	

<table>
<tr><td></td><td>$\frac{dy}{dx} = \frac{3}{3x-1}$
When $x = 7$, $\frac{dy}{dx} = \frac{3}{3(7)-1} = 0.15$
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
$0.06 = 0.15 \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 0.4$
$\therefore$ When $x = 7$, x-coordinate is increasing at a rate of 0.4 units per second.</td><td>[1]

[2]</td></tr>
<tr><td>8(b)(i)</td><td>$y = 8 - (2x+1)^3$
$\frac{dy}{dx} = -3(2x+1)^2(2) = -6(2x+1)^2$
At stationary points, $\frac{dy}{dx} = 0$.
$\Rightarrow -6(2x+1)^2 = 0$
Since the equation $(2x+1)^2 = 0$ has real and equal roots, $\therefore$ the curve has only one stationary point.
$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$
When $x = -\frac{1}{2}$, $y = 8$.
<table>
<tr><td>x</td><td>$\left(-\frac{1}{2}\right)^-$</td><td>$\left(-\frac{1}{2}\right)$</td><td>$\left(-\frac{1}{2}\right)^+$</td></tr>
<tr><td>Sign of $\frac{dy}{dx}$</td><td>–ve</td><td>0</td><td>–ve</td></tr>
<tr><td>Slope of tangent</td><td>\</td><td>—</td><td>\</td></tr>
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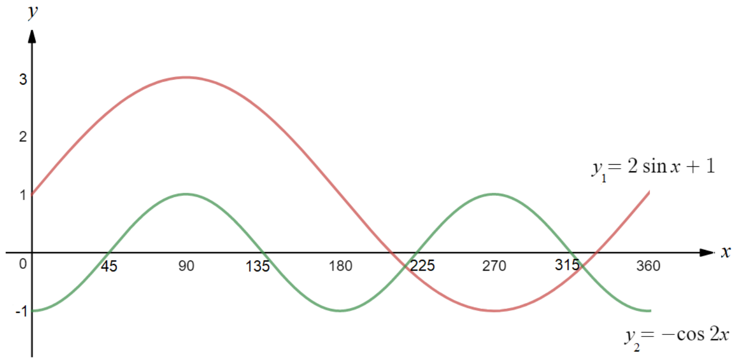
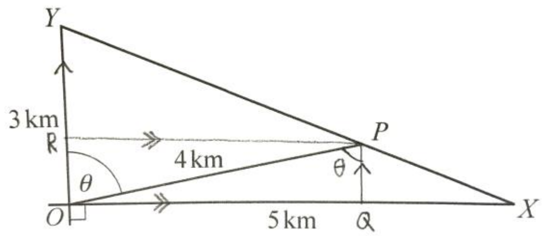
$\therefore$ The stationary point $\left(-\frac{1}{2}, 8\right)$ is a point of inflexion.</td><td>[1]

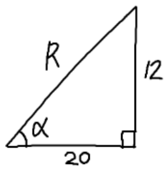
[1]
[1]

[2] only 1 mark if workings showed only $\frac{d^2y}{dx^2} = 0$.</td></tr>
<tr><td>8(b)(ii)</td><td>$\therefore$ Coordinates of the stationary point is $\left(-\frac{1}{2}, 8\right)$.</td><td>[1]</td></tr>
<tr><td>9(i)</td><td>$\therefore$ Gradient of $AB = \frac{p-1}{0-(-2)} = \frac{p-1}{2}$
$\therefore$ Gradient of $CB = \frac{p-3}{0-1} = 3 - p$
Since $\angle ABO = \angle CBO$, gradient of $AB = -$gradient of CB.
$\Rightarrow \frac{p-1}{2} = -(3-p)$
$p - 1 = 2p - 6$
$\therefore$ $p = 6 - 1 = 5$ (shown)</td><td>[1]
[1]

[1]
[1]</td></tr>
<tr><td>9(ii)</td><td>Gradient of $AB = \frac{5-1}{2} = 2$</td><td></td></tr>
</table>

	Gradient of $DC = 2$ (AB is parallel to DC.) Equation of DC is $y - 3 = 2(x - 1) \Rightarrow y = 2x + 1$ -(1) Since $AB \perp AD$, gradient of $AD = -\frac{1}{2}$ Equation of AD is $y - 1 = -\frac{1}{2}[x - (-2)] \Rightarrow y = -\frac{1}{2}x$ -(2) Substitute -(1) into -(2), $2x + 1 = -\frac{1}{2}x \Rightarrow x = -\frac{2}{5}$ Substitute $x = -\frac{2}{5}$ into $y = 2x + 1$, $y = 2\left(-\frac{2}{5}\right) + 1 = \frac{1}{5}$ $\therefore$ Coordinates of D is $\left(-\frac{2}{5}, \frac{1}{5}\right)$.	[2] [2] [1] [1]
9(iii)	$\therefore$ Area of trapezium $ABCD$ $= \frac{1}{2}\begin{vmatrix} 0 & -2 & -\frac{2}{5} & 1 & 0 \\ 5 & 1 & \frac{1}{5} & 3 & 5 \end{vmatrix}$ $= \frac{1}{2}\left(-\frac{2}{5} - 1\frac{1}{5} + 5\right) - \frac{1}{2}\left(-10 - \frac{2}{5} + \frac{1}{5}\right)$ $= 6\frac{4}{5}$ units^2	 [1] [1]
10(i)(a)	$y_1 = 2\sin x + 1$ $\therefore$ Amplitude = 2, period = 360°	 [1]
10(i)(b)	$y_2 = -\cos 2x$ $\therefore$ Amplitude = 1, period = 180°	 [1]
10(ii)	$y_1 = y_2$ $2\sin x + 1 = -\cos 2x$ $2\sin x + 1 + \cos 2x = 0$ $2\sin x + 1 + 1 - 2\sin^2 x = 0$ $\sin^2 x - \sin x - 1 = 0$ $\sin x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2}$ $\sin x = \frac{1+\sqrt{5}}{2}$ (N.A. $\because -1 \le \sin x \le 1$) or $\sin x = \frac{1-\sqrt{5}}{2}$ $\Rightarrow \sin x = \frac{1-\sqrt{5}}{2}$ basic $\angle = \sin^{-1}\left(\left\lvert\frac{1-\sqrt{5}}{2}\right\rvert\right) = 38.17°$ (2 d.p.) For $0° \le x \le 360°$, $x = 180° + 38.17°, 360° - 38.17°$ $\therefore x = 218.2°$ (1 d.p.), $321.8°$ (1 d.p.)	 [1] [1] [1] [1]

10(iii)		y_1: [1] shape, equation labelled; [1] period, *y*-intercept y_2: [1] shape, equation labelled; [1] axial intercepts
10(iv)	For $y_1 - y_2 > 0$, $y_1 > y_2 \Rightarrow 2\sin x + 1 > -\cos 2x$ From solutions in 10(ii) and sketch in 10(iii), $\therefore$ set of values of x is $0° \le x < 218.2°$ or $321.8° < x \le 360°$.	[2]
11(i)	 Let Q and R be the foot of perpendicular from P to OX and OY respectively. $\Rightarrow PQ \,//\, YO$ and $RP \,//\, OX$ $\angle YOP = \angle OPQ = \theta$ (alternate $\angle$s) $\dfrac{PQ}{OP} = \cos\theta$ $\Rightarrow PQ = OP\cos\theta = 4\cos\theta$ $\therefore$ Shortest distance, in km, of P from $OX = 4\cos\theta$ $\dfrac{RP}{OP} = \sin\theta$ $\Rightarrow RP = OP\sin\theta = 4\sin\theta$ $\therefore$ Shortest distance, in km, of P from $OY = 4\sin\theta$	[½] [½]
11(ii)	Area of ΔOPX + Area of ΔOPY = Area of ΔOXY $\frac{1}{2}\times PQ \times OX + \frac{1}{2}\times RP \times YO = \frac{1}{2}\times OX \times OY$ $\frac{1}{2}(4\cos\theta)(5) + \frac{1}{2}(4\sin\theta)(3) = \frac{1}{2}\times 5 \times 3$ $\frac{1}{2}\times 20\cos\theta + \frac{1}{2}\times 12\sin\theta = \frac{1}{2}\times 15$ $\therefore 20\cos\theta + 12\sin\theta = 15$ (shown)	[2] [1]
11(iii)	$20\cos\theta + 12\sin\theta = R\cos(\theta - \alpha)$ $\frac{20}{R}\cos\theta + \frac{12}{R}\sin\theta = \cos\theta\cos\alpha + \sin\theta\sin\alpha$ Comparing terms on both sides, $\cos\alpha = \dfrac{20}{R}$ and $\sin\alpha = \dfrac{12}{R}$.	[1]

	 By Pythagoras Theorem, $R^2 = 12^2 + 20^2 = 544$. $R = \sqrt{544} = 4\sqrt{34}$ $\alpha = \tan^{-1}\left(\frac{12}{20}\right) \approx 30.96° = 31.0°$ (1 d.p.) $\therefore 20\cos\theta + 12\sin\theta = 4\sqrt{34}\cos(\theta - 31.0°)$	[2] [1]
11(iv)	$20\cos\theta + 12\sin\theta = 15$ $\Rightarrow 4\sqrt{34}\cos(\theta - 31.0°) = 15$ $\cos(\theta - 31.0°) = \frac{15}{4\sqrt{34}}$ $\theta - 31.0° = \cos^{-1}\left(\frac{15}{4\sqrt{34}}\right) = 49.98°$ (2 d.p.) $\therefore \theta = 49.98° + 30.96° = 80.9°$ (1 d.p.)	[1] [1]