1 A curve is such that $\frac{dy}{dx} = 5 - \sqrt{x-3}$ and A (7, 15) is a point on the curve.

(i) Find the equation of the curve. [2]

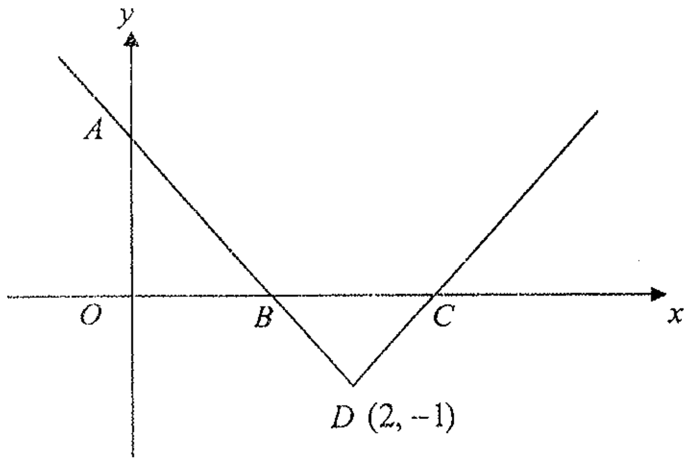
(ii) Find the equation of the normal to the curve at A. [2]

2 The equation of a curve is given by $y = x^2 + 2ax + 2a - 3$, where a is a constant. Show that, for all values of a, the curve intersects the x-axis at two distinct points. [4]

3 The diagram shows the graph of $y = |p - 2x| + q$, where p and q are integers. A is the point where the graph intersects the y-axis, and B and C are the points where it intersects the x-axis. Point $D(2, -1)$ is the vertex of the graph.

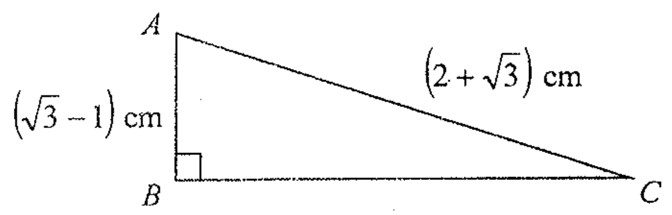
(i) Find the value of p and of q. [2]

(ii) Hence find the coordinates of A, B and C. [4]



4 (i) Express $\left(\frac{1-\sqrt{3}}{2+\sqrt{3}}\right)^2$ in the form $a + b\sqrt{3}$ where a and b are integers. [3]

(ii) The diagram shows a triangle ABC where angle $ABC = 90°$, $AB = (\sqrt{3}-1)$ cm and $AC = (2+\sqrt{3})$ cm. Using your answer to **(i)**, find the exact value of $\cos^2(A\hat{C}B)$ without using a calculator. [3]



5 It is given that point A (8, 11) lies on the line l with equation $y = 2x - 5$, and P is the point (1, 2).

If B is the point on l such that PBA is a right-angled triangle, find

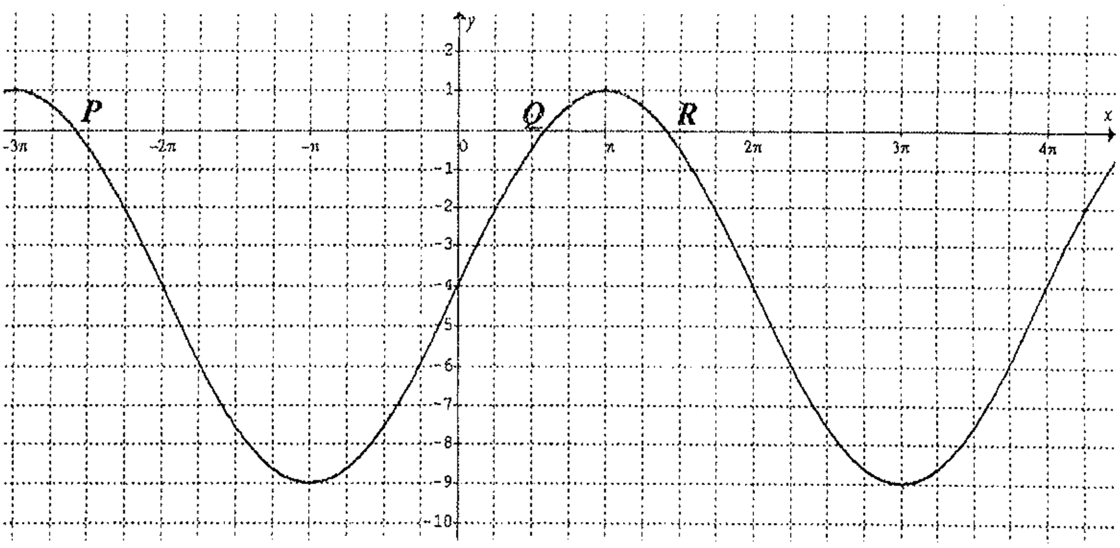
(i) the coordinates of B, [5]

(ii) the area of triangle PBA. [2]

6 (i) Prove that $(\sin 2y + 2)(\sin y - \cos y) = 2\cos^3 y\,(\tan^3 y - 1)$. [4]

(ii) Hence find the acute angle y, in degrees, such that $(\sin 2y + 2)(\sin y - \cos y) = 2\cos^3 y$. [2]

7 The figure shows part of the graph of $y = a\sin(bx) + c$. Points P, Q and R on the graph lie on the x axis.



(i) Find the value of each of the constants a, b and c. [3]

(ii) Hence find the coordinates of P, Q and R. [4]

8 A curve has the equation $y = 4-(3x-1)^2$.

(i) Explain why the highest point on the curve has coordinates $\left(\frac{1}{3}, 4\right)$. [1]

(ii) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(iii) Sketch the graph of $y = \left|4-(3x-1)^2\right|$. [2]

(iv) The equation $\left|4-(3x-1)^2\right| = mx+4$ has 4 distinct solutions. Using your graph, determine the range of values of m. [2]

9 (i) Show that $\frac{d}{dx}\left(x\cos^2 3x\right) = \cos^2 3x - 3x\sin 6x$. [3]

(ii) Hence integrate $x\sin 6x$ with respect to x. [4]

10 (i) Sketch the parabola $y^2 = 2x$. [2]

(ii) The curve $y^2 = 2x$ intersects the straight line $y = 3x-1$ at the points A and B. Find the coordinates of the midpoint of AB. [6]

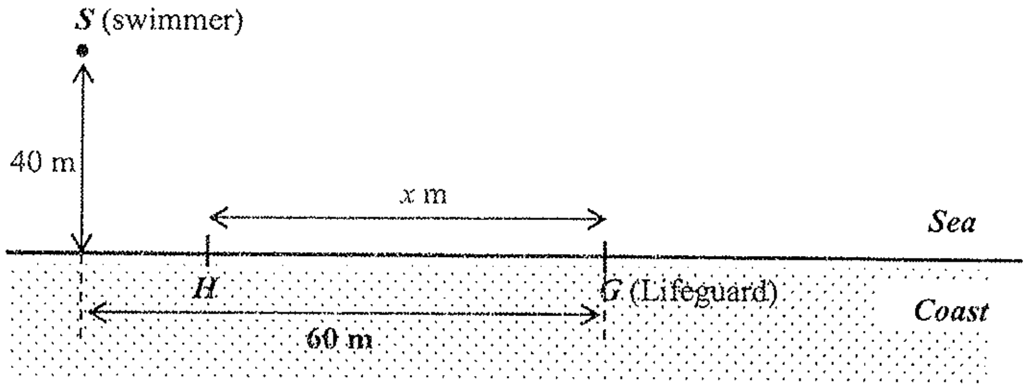
11 (i) Express $\frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}$ in partial fractions. [5]

(ii) Hence show that

$$\int_1^2 \frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}\,dx = \frac{83}{42} + \frac{\ln 27}{2} + \ln\left(\frac{49}{36}\right).$$ [4]

12 A lifeguard at a beach resort is stationed at point G along the coastline, as shown in the diagram below. When he detects a swimmer who needs help at a point S, he would run along the coastline over a distance of x m to a point H, and then swim in a straight line, HS, towards the swimmer. The lifeguard runs at a speed of 4 m/s and swims at a speed of 2 m/s.

A swimmer in distress is detected at a position that is 40 m away from the coastline, and the foot of perpendicular from the swimmer to the coastline is at a distance of 60 m away from the lifeguard.



(i) Show that the time taken by the lifeguard to swim from H to S is $\dfrac{\sqrt{1600+(60-x)^2}}{2}$ seconds. [2]

(ii) Find, in terms of x, the total time T taken by the lifeguard to reach the swimmer. [1]

(iii) Obtain an expression for $\dfrac{dT}{dx}$. [2]

(iv) Find the value of x such that the lifeguard would be able to reach the swimmer in the shortest possible time. [4]

END OF PAPER

ANDERSON SECONDARY SCHOOL
Preliminary Examination 2017
Secondary Four Express and Five Normal Academic

ADDITIONAL MATHEMATICS PAPER 1
ANSWERS

1 (i) $3y = 15x - 2(x-3)^{\frac{3}{2}} - 44$ or $y = 5x - \frac{2}{3}(x-3)^{\frac{3}{2}} - \frac{44}{3}$

(ii) $3y = 52 - x$

3 (i) $p = 4$, $q = -1$

(ii) $A = (0,3)$, $B = \left(\frac{3}{2}, 0\right)$, $C = \left(\frac{5}{2}, 0\right)$

4 (i) $52 - 30\sqrt{3}$ (ii) $30\sqrt{3} - 51$

5 Case 1: right-angle at B

(i) $(3, 1)$ (ii) 12.5 units^2

Case 2: right-angle at P

(i) $(2\frac{4}{5}, \frac{3}{5})$ (ii) 13 units^2

6 (ii) $y = 51.6°$ (1dp)

7 (i) $a = 5$, $b = 0.5$, $c = -4$

(ii) $P = (-8.14, 0)$, $Q = (1.85, 0)$ and $R = (4.32, 0)$

8 (ii) $(1, 0)$ and $\left(-\frac{1}{3}, 0\right)$ (iv) $-4 < m < 0$

9 (ii) $\int x \sin 6x \, dx = \frac{x}{6} + \frac{\sin 6x}{36} - \frac{x\cos^2 3x}{3} + c$

10 (ii) $\left(\frac{4}{9}, \frac{1}{3}\right)$

11 (i) $2 + \frac{3}{2x-1} + \frac{2}{x+5} - \frac{1}{(x+5)^2}$

12 (ii) $\frac{\sqrt{1600 + (60-x)^2}}{2} + \frac{x}{4}$ seconds

(iii) $\frac{dT}{dx} = \frac{x - 60}{2\sqrt{1600 + (60-x)^2}} + \frac{1}{4}$

(iv) $x = 36.9$