1 (i) Given that the coefficient of x^2 in the expansion of $(1-2x)^2(1+px)^7$ is 32, find the possible values of the constant p. [4]

(ii) Hence find the coefficient of x^2 in the expansion of $(1-x)^2\left(1+\frac{px}{2}\right)^7$. [2]

2 (i) Using $\sin 3x \equiv \sin(2x+x)$, show that $\sin 3x$ may be expressed as $3\sin x - 4\sin^3 x$. [3]

(ii) Hence solve the equation $\sin 3x = 2\sin x$ for $0 < x < 2\pi$. [5]

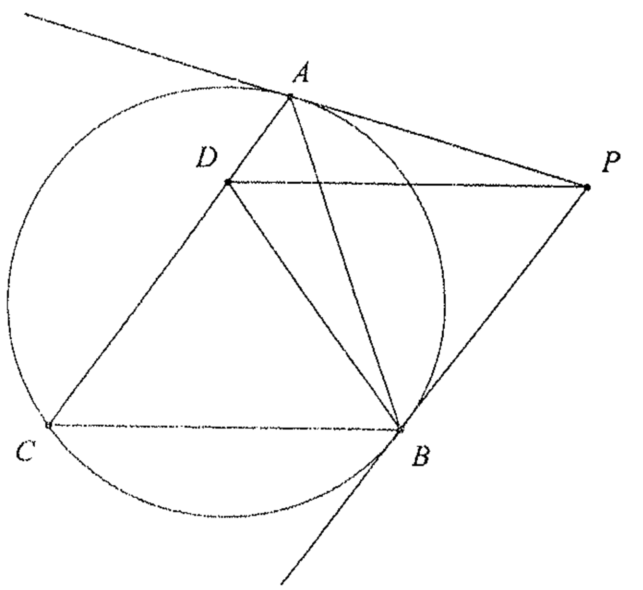
3 The roots of a quadratic equation $x^2 = 12x - 4$ are α^2 and β^2 where $\alpha < 0, \beta < 0$ and $\alpha < \beta$. Find, without calculating the value of α and β,

(i) the value of $\alpha\beta$, [2]

(ii) the value $\alpha + \beta$, [3]

(iii) a quadratic equation with roots $\alpha + \beta$ and $\alpha - \beta$, in the form $(x+a)(x+b)=0$ where a and b are real numbers. [3]

4



The diagram shows a circle passing through the vertices of a triangle ABC. The tangents to the circle at A and B intersect at the point P. The point D lies on AC such that PD is parallel to BC. Prove that

(i) angle $ADP =$ angle ABP, [2]

(ii) A, D, B and P lie on a circle, [1]

(iii) $DB = DC$. [4]

5 It is given that $y = \frac{1+2x}{e^{3x}}$.

(i) Obtain an expression for $\frac{dy}{dx}$ in the form $\frac{ax+b}{e^{3x}}$, where a and b are integers. [2]

(ii) Determine the range of values of x for which y is decreasing. [3]

The values of x and y are such that, when $x = 1$, y is increasing at a rate of $\frac{1}{4}$ units per second.

(iii) Find the exact rate of change of x when $x = 1$. [3]

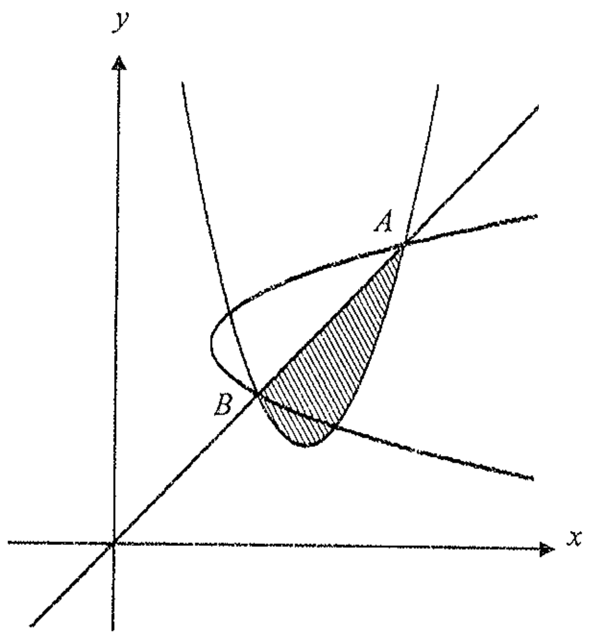
6 It is given that $f(x) = 2x^3 + x^2 - 8x - 4$.

(i) Factorise $f(x)$, showing your working clearly. [3]

(ii) Show that the equation $f(x) + 10x + 5 = 0$ has only one real root and state its value. [3]

(iii) Find the range of value of the constant k for which the graph of $y = f(x) + kx$ has two stationary points. [4]

7



The diagram shows part of the graphs of $y = 3x^2 - 8x + 6$, $x = 3y^2 - 8y + 6$ and the line $y = x$, intersecting at the points A and B.

(i) Find the x-coordinates of A and B. [3]

(ii) Find the area of the shaded region bounded by the curve $y = 3x^2 - 8x + 6$ and the line $y = x$. [3]

(iii) Deduce the area of the region bounded by the curve $x = 3y^2 - 8y + 6$ and the line $y = x$. [2]

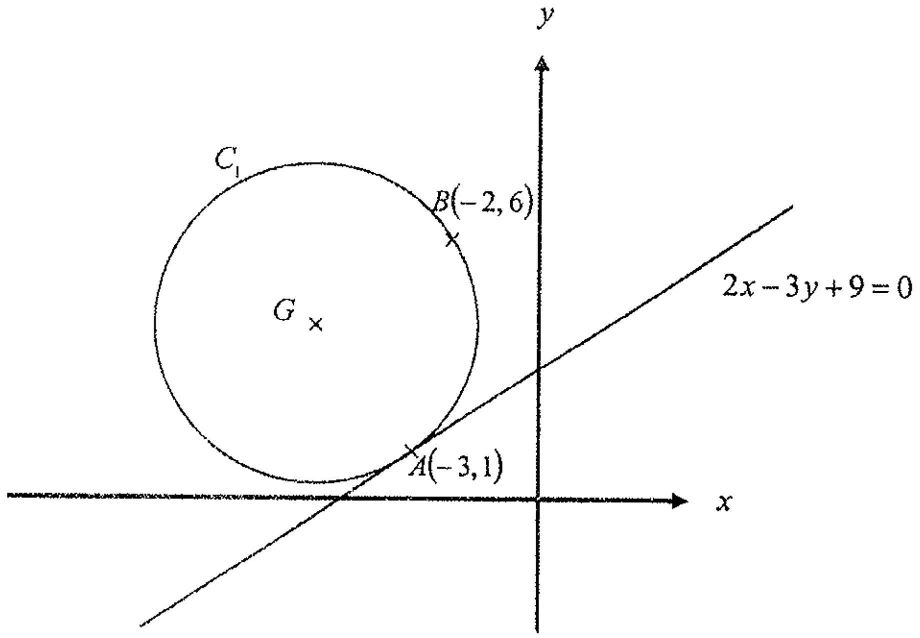
8 A point P moves along a straight line such that its displacement, x cm, t seconds after leaving a fixed point, is given by $x = 5\cos 2t - 6\sin t$. Find

(i) the exact velocity of P when $t = \frac{\pi}{6}$, [3]

(ii) the value of t between 0 and π for which P is instantaneously at rest, [4]

(iii) the distance travelled by P in the first 2 seconds. [4]

9



In the diagram, the circle C_1 with centre G passes through the point $B(-2, 6)$ and touches the line $2x - 3y + 9 = 0$ at the point $A(-3, 1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

(ii) Show that G is $(-5, 4)$. [4]

(iii) Find the equation of the circle C_1. [2]

A second circle, C_2 with centre H is a reflection of the circle C_1 along the line $2x - 3y + 9 = 0$. Find

(iv) the coordinates of H, [2]

(v) the equation of circle C_2. [1]

10 (a) (i) Explain why the equation $\sqrt{3-e^x}+1-ke^x=0$ has no solution if $k<0$. [3]

(ii) Solve the equation $3-\sqrt{3-e^x}=e^x$. [3]

(b) In a chemical reaction, the mass, x kg, of a certain substance produced after t hours is given by the equation $\ln\left(\frac{ax}{1-x}\right)=bt$ where a and b are positive constants to be determined.

(i) Given that the initial mass of the substance is $\frac{1}{5}$ kg and $x=\frac{1}{3}$ when $t=1$, determine the exact value of a and of b. [4]

(ii) Express x in terms of t. [2]

11 Mr Lee bought a car on 1st January 2013. The market value of the car decreases each year from 2013 to 2017. He estimated that the value $\$V$ of his car t years after 1st January 2013 can be modelled by the equation

$$V=V_0a^t,$$

where V_0 and a are constants. The table below gives values of V and t.

t	1	2	3	4
V	81 000	72 900	65 600	59 000

(i) Plot a suitable straight line graph for $4.5<\lg V<5.0$ and show that the model is valid from 2013 to 2017. [4]

(ii) Estimate the value of V_0 and a. Hence deduce the price Mr Lee paid for the car on 1 January 2013. [5]

(iii) Assuming that the model is still appropriate, estimate the market value of the car on 1st January 2018. [1]

End of Paper

Secondary 4E5N
Preliminary Examination 2017
ADDITIONAL MATHEMATICS
Answer Keys for Paper 2

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1 (i) $p = -\frac{2}{3}$ or 2

(ii) 8

2 (i) Proof

(ii) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

3 (i) $\alpha\beta = 2$

(ii) $\alpha + \beta = -4$

(iii) $\alpha - \beta = -\sqrt{8}$
$(x+4)(x+2\sqrt{2}) = 0$

4 (i) Use corresponding $\angle$ s, $PD \mathbin{//} BC$ **AND** $\angle$ s in alternate segment.

(ii) Use the result of (i).

(iii) Use $\angle$ s in the same segment, alternate $\angle$ s, $PD \mathbin{//} BC$ **AND** $\angle$ s in alternate segment.

5 (i) $-\frac{6x+1}{e^{3x}}$

(ii) $x > -\frac{1}{6}$

(iii) $-\frac{e^3}{28}$ units/s

6 (i) $(2x+1)(x+2)(x-2)$

(ii) $(2x+1)(x^2+1) = 0$
Since $x^2 + 1 > 0$, $2x + 1 = 0$.
$\therefore$ The equation has only one solution i.e. $x = -\frac{1}{2}$.

(iii) $k < 8\frac{1}{6}$

7 (i) x-coordinates of A and B are 2 and 1 respectively.

(ii) $\frac{1}{2}$ units^2

(iii) The curve $x = 3y^2 - 8y + 6$ is a **reflection (or mirror image)** of the curve $y = 3x^2 - 8x + 6$ in the line $y = x$. $\therefore$ the area bounded by the curve $x = 3y^2 - 8y + 6$ and the line $y = x$ is also $\frac{1}{2}$ units^2.

8 (i) $-8\sqrt{3}$ cm/s

(ii) $t = \frac{\pi}{2}$,

(iii) 18.3 cm

9 (i) $y = -\frac{1}{5}x + 3$

(ii) Equation of AG is $y = -\frac{3}{2}x - \frac{7}{2}$

Coordinates of $G = (-5, 4)$

(iii) $x^2 + y^2 + 10x - 8y + 28 = 0$

(iv) Coordinates of $H = (-1, -2)$.

(v) Equation of circle C_2 is $(x+1)^2 + (y+2)^2 = 13$

10 (a) (i) Show that $\sqrt{3 - e^x} + 1 - ke^x \neq 0$ or $\sqrt{3 - e^x} + 1 \neq ke^x$

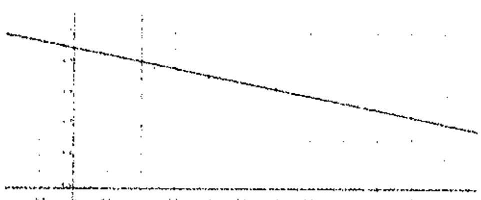
(ii) $\ln 3$ or $\ln 2$

(b) (i) $a = 4, b = \ln 2$

(ii) $x = \frac{2^t}{4 + 2^t}$

11 (i) $\lg V = (\lg a)t + \lg V_0$

t	1	2	3	4
$\lg V$	4.91	4.86	4.82	4.77



(ii) $a = 0.900$ (3sf), $V_0 = 90100$(3sf). Mr Lee paid \$90100 for the car.

(iii) \$53200