Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2017
SECONDARY 4

ADDITIONAL MATHEMATICS — 4047/01

Paper 1

28 August 2017

2 hours

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

/ 80

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The equation of a curve is $y = 2x^2 + (k+2)x - k$, where k is a constant.
Find the values of k for which the line $y = 4x - 2$ is a tangent to the curve. [3]

2 The volume of a right circular cone is 2π m^3. The radius of its base is $(1+\sqrt{3})$ m.
Find, without using a calculator, the height of the cone in the form $a + b\sqrt{3}$, where a and b are integers. [4]

3 **(i)** Sketch the curve $y^2 = x - 2$. [2]

(ii) The curve $y^2 = x - 2$ intersects the straight line $x + 3y = 6$ at two points P and Q.
Find the distance PQ. [5]

4 Express $\dfrac{5x^2+4}{(x+2)(x^2-1)}$ in partial fractions. [5]

5 It is given that $\mathrm{f}(x)$ is such that $\mathrm{f}'(x) = \dfrac{1}{(5x-2)^3}$. Given also that $\mathrm{f}(0.5) = 1$, show that [5]

$$3\,\mathrm{f}''(1) + 2\,\mathrm{f}(1) = \frac{20}{9}.$$

6 **(i)** Sketch the graph of $y = |4x - x^2|$, showing clearly the coordinates of the maximum point and of the points where the curve meets the x-axis. [2]

A line of gradient m passes through the point $(0, c)$.

(ii) In the case where $m = 4$ and $c = 9$, find the x-coordinates of the points of intersection between the line and the graph of $y = |4x - x^2|$. [3]

(iii) In the case where $m = 0$, determine the set of values of c for which the line intersects the graph of $y = |4x - x^2|$ at two points. [2]

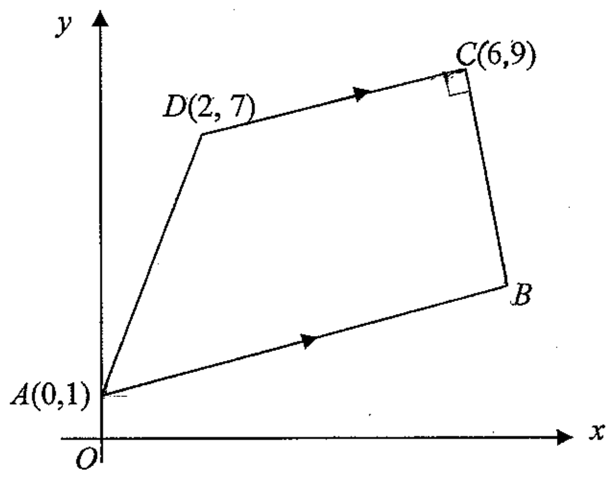
7 The equation of a curve is $y = x\ln 3x$, where $x > 0$.

(i) Find $\frac{dy}{dx}$. [1]

(ii) Hence, evaluate $\frac{2}{5}\int_1^3 \ln 3x \, dx$. [4]

(iii) Find the range of values of x for which the function $y = x\ln 3x$ is increasing, leaving your answer in terms of e. [2]

8 **Solutions to this question by accurate drawing will not be accepted.**

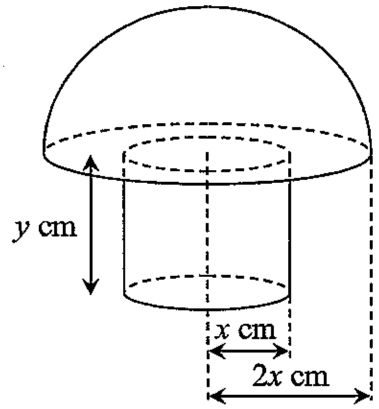


The diagram shows a trapezium $ABCD$, with $A(0,1)$, $C(6,9)$ and $D(2,7)$.
AB is parallel to DC and angle $BCD = 90°$.

(i) Find the coordinates of B. [4]

(ii) E is the point $(5, y)$, that lies above the line AB, such that the area of triangle ABE is twice the area of trapezium $ABCD$.
Find the coordinates of E. [3]

9 The diagram shows a solid which consists of a hemisphere fixed on top of a cylinder.
The cylinder has a radius of x cm and a height of y cm.
The hemisphere has a radius of $2x$ cm.



(i) Given that the total volume of the solid is 100π cm^3, express y in terms of x. [2]

(ii) Show that the total surface area, A cm^2, of the solid is given by

$$A = 20\pi\left(\frac{10}{x}+\frac{x^2}{15}\right).$$ [2]

(iii) Given that x can vary, find the value of x for which A has a stationary value and determine whether this stationary value is a maximum or minimum. [4]

10 The table shows some experimental values of two variables x and y. One of the values of y is incorrectly recorded.

x	0.5	1.0	2.5	3	4
y	1.33	1.14	0.95	0.73	0.62

It is known that x and y are related by the equation $y = \frac{a}{x+b}$, where a and b are constants.

(i) Plot $\frac{1}{y}$ against x and hence determine the value of y which is incorrectly recorded. [2]

(ii) Draw the straight line graph and use it to estimate the correct value of y. [2]

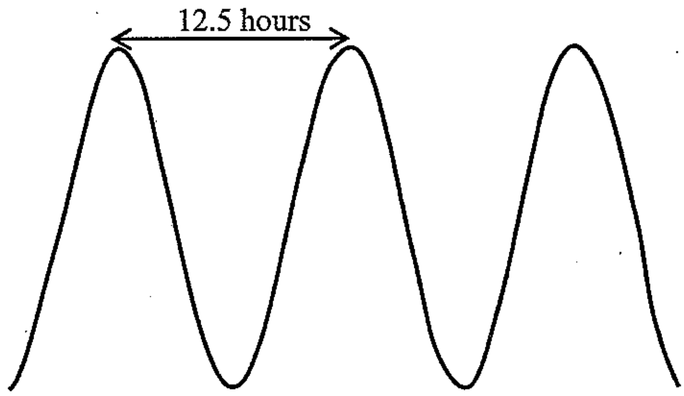
(iii) Use your graph to estimate the value of a and of b. [3]

(iv) By drawing another straight line on the graph in **(ii)**, solve the simultaneous equations $y = \frac{a}{x+b}$ and $\frac{1}{y}+\frac{1}{2}x = 2$. [2]

11 The height of the water tides in Singapore, h metres, is modelled by the equation $h = 1.1\sin kt + 1.8$, where k is a constant, and t is the time in hours after midnight.

The diagram below shows the model of the height of the tides.

The average time difference between two successive high tides is 12.5 hours.



(i) Explain why this model suggests that the highest tide is 2.9 m. [1]

(ii) Show that the value of k is $\dfrac{4\pi}{25}$. [2]

(iii) A lifeguard starts his patrol duty shift when the height of the tides is 2.5 m or more. Find the duration of his patrol duty shift. [4]

12 A curve is such that $\dfrac{d^2y}{dx^2} = 12x + p$ where p is a constant. The tangent to the curve at $x = 3$ is parallel to the x-axis. The curve passes through the point (1, 12) and at this point the gradient of the curve is −4.

(i) Find the value of p and hence determine the equation of the curve. [7]

(ii) Find the minimum **gradient** of the curve and the value of x when the minimum gradient occurs. [4]

END OF PAPER