2017 CCHM 84 AM Prelim P1

1. $y = 4x - 2$ ①

$y = 2x^2 + (k+2)x - k$ ②

① → ②: $2x^2 + (k+2)x - k = 4x - 2$.

$2x^2 + (k-2)x - (k-2) = 0$.

Given that ① is a tangent to ②: Discriminant $= 0$.

$(k-2)^2 - 4(2)(-1)(k-2) = 0$.

$k^2 - 4k + 4 + 8k - 16 = 0$.

$k^2 + 4k - 12 = 0$.

$(k+6)(k-2) = 0$.

$\therefore k = -6$ or 2

2. $\frac{1}{3}\pi(1+\sqrt{3})^2 h = 2\pi$, where h is the height of the cone.

$(1 + 2\sqrt{3} + 3)h = 6$.

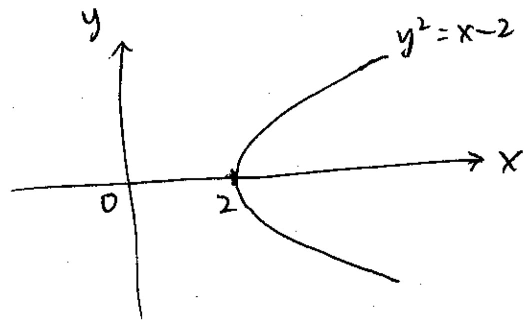
$\therefore h = \dfrac{6}{4 + 2\sqrt{3}}$

$= \dfrac{3}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{6 - 3\sqrt{3}}{4-3}$

$= 6 - 3\sqrt{3}$, where $a = 6$, $b = -3$.

3. (i)



3. (ii) $x+3y=6.$ ①

$y^2=x-2$ ②

① → ②: $y^2=6-3y-2$

$y^2+3y-4=0.$

$(y+4)(y-1)=0.$

$y=-4$ or 1

When $y=-4$, $x=6-3(-4)=18.$

When $y=1$, $x=6-3(1)=3.$

Let P be $(3,1)$ and Q be $(18,-4)$.

$\therefore$ Distance $PQ=\sqrt{(18-3)^2+(-4-1)^2}$

$=\sqrt{250}$

$=5\sqrt{10}$ units.

4. $\dfrac{5x^2+4}{(x+2)(x^2-1)}=\dfrac{5x^2+4}{(x+2)(x-1)(x+1)}$

Let $\dfrac{5x^2+4}{(x+2)(x-1)(x+1)}=\dfrac{A}{x+2}+\dfrac{B}{x-1}+\dfrac{C}{x+1}$ for some constants, A, B, C

$5x^2+4=A(x-1)(x+1)+B(x+2)(x+1)+C(x+2)(x-1)$

When $x=1$, $5(1)^2+4=B(3)(2)$

$B=\frac{3}{2}.$

When $x=-1$, $5(-1)^2+4=C(1)(-2)$

$C=-\frac{9}{2}.$

When $x=-2$, $5(-2)^2+4=A(-3)(-1)$

$A=8.$

$\therefore \dfrac{5x^2+4}{(x+2)(x^2-1)}=\dfrac{8}{x+2}+\dfrac{3}{2(x-1)}-\dfrac{9}{2(x+1)}.$

5. $f'(x) = \frac{1}{(5x-2)^3}$.

$$f(x) = \int f'(x)\, dx$$
$$= \int \frac{1}{(5x-2)^3}\, dx$$
$$= \frac{(5x-2)^{-3+1}}{(-2)(5)} + c \quad \text{for some constant } c$$
$$= -\frac{1}{10(5x-2)^2} + c.$$

Given that $f(0.5) = 1$,

$$-\frac{1}{10(\frac{5}{2}-2)^2} + c = 1.$$
$$c = \frac{7}{5}.$$

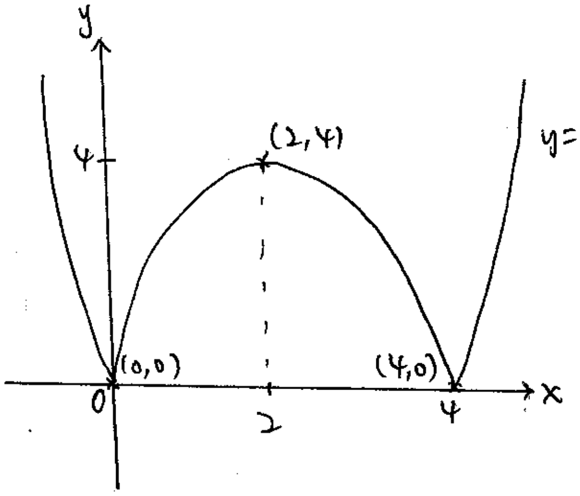
$$f(x) = -\frac{1}{10(5x-2)^2} + \frac{7}{5}.$$

$$f''(x) = \frac{d}{dx}\left(\frac{1}{(5x-2)^3}\right)$$
$$= -\frac{3}{(5x-2)^4}(5)$$
$$= -\frac{15}{(5x-2)^4}$$

$\therefore$ $3f''(1) + 2f(1)$

$$= 3\left[-\frac{15}{(5-2)^4}\right] + 2\left[-\frac{1}{10(5-2)^2} + \frac{7}{5}\right]$$
$$= -\frac{5}{9} + \frac{25}{9}$$
$$= \frac{20}{9} \quad \text{(shown)}.$$

6. (i)



6. (ii)

$y=4x+9$ ①

$y=|4x-x^2|$ ②

① → ②: $|4x-x^2|=4x+9$.

$4x-x^2=4x+9$ or $4x-x^2=-4x-9$

$x^2+9=0$. $\qquad x^2-8x-9=0$.

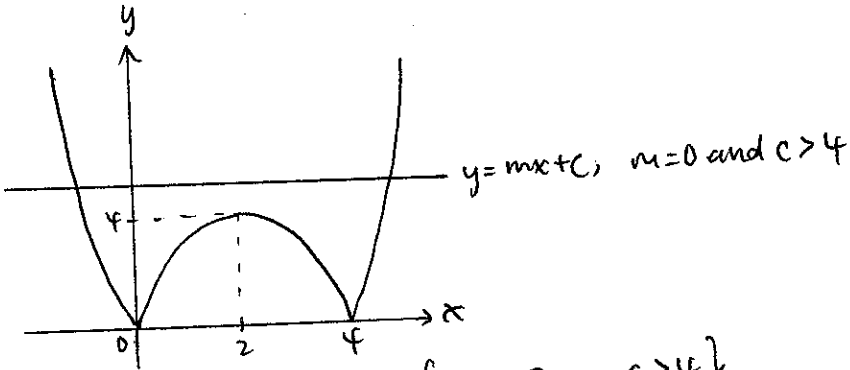
$x^2=-9$ $\qquad (x+1)(x-9)=0$.

Since $x^2 \geqslant 0$, $\qquad x=-1$ or 9.

$x^2=-9$ has no solution.

$\therefore$ x-coordinates of points of intersection are $x=-1$ and $x=9$.

6. (iii)



$\therefore$ set of values of c is $\{c : c=0 \text{ or } c>4\}$

7. (i) $y = x\ln 3x$, $x>0$.

$\therefore \frac{dy}{dx} = \ln 3x + x\left(\frac{1}{3x}\right)(3)$

$= \ln 3x + 1$.

7. (ii) $\int_1^3 \ln 3x + 1 \, dx = \left[x \ln 3x\right]_1^3$

$\int_1^3 \ln 3x \, dx + \left[x\right]_1^3 = 3\ln 9 - \ln 3.$

$\int_1^3 \ln 3x \, dx = 3 \ln 9 - \ln 3 - 2$

$= \ln 243 - 2.$

$\therefore \frac{2}{5}\int_1^3 \ln 3x \, dx = \frac{2}{5}\ln 243 - \frac{4}{5}$

$= \ln 243^{\frac{2}{5}} - \frac{4}{5}$

$= \ln 9 - \frac{4}{5}$ or 1.40 (3 s.f.)

7. (iii) For y to be increasing, $\frac{dy}{dx} > 0$.

$\ln 3x + 1 > 0.$

$\ln 3x > -1.$

$3x > \frac{1}{e}$

$\therefore x > \frac{1}{3e}$

8. (i) Let B be (x_B, y_B).

Gradient of AB = gradient of DC ($AB // DC$)

$\frac{y_B - 1}{x_B - 0} = \frac{9-7}{6-2}$

$y_B = \frac{1}{2}x_B + 1$ ①

Gradient of DC × Gradient of $BC = -1$ ($DC \perp CB$)

Gradient of $BC = \frac{-1}{\left(\frac{1}{2}\right)} = -2.$

$\frac{9 - y_B}{6 - x_B} = -2.$

$9 - y_B = -12 + 2x_B$ ②

① → ②: $9 - \frac{1}{2}x_B - 1 = -12 + 2x_B.$

$\frac{5}{2}x_B = 20.$

$\Rightarrow x_B = 8.$

$\Rightarrow y_B = \frac{1}{2}(8) + 1 = 5.$

$\therefore B$ is $(8, 5)$.

8. (ii) Area of trapezium ABCD $= \frac{1}{2}\begin{vmatrix} 2 & 0 & 8 & 6 & 2 \\ 7 & 1 & 5 & 9 & 7 \end{vmatrix}$

$= \frac{1}{2}(2+72+42) - \frac{1}{2}(8+30+18)$

$= 30$ units^2.

$\Rightarrow$ Area of $\triangle ABE = 60$ units^2.

$\frac{1}{2}\begin{vmatrix} 0 & 8 & 5 & 0 \\ 1 & 5 & y & 1 \end{vmatrix} = 60.$

$\frac{1}{2}(8y+5) - \frac{1}{2}(8+25) = 60.$

$8y + 5 - 8 - 25 = 120.$

$8y = 148.$

$y = 18\frac{1}{2}$

$\therefore E$ is $(5, 18\frac{1}{2})$

9. (i) Volume of hemisphere + Volume of cylinder $= 100\pi$

$\frac{1}{2}\cdot\frac{4}{3}\pi(2x)^3 + \pi(x)^2(y) = 100\pi$

$\frac{16}{3}\pi x^3 + \pi x^2 y = 100\pi.$

$x^2 y = 100 - \frac{16}{3}x^3$

$\therefore y = \frac{100}{x^2} - \frac{16}{3}x.$

9. (ii) Exposed surface area of hemisphere

$= 2\pi(2x)^2 + \pi(2x)^2 - \pi(x)^2$

$= 11\pi x^2.$

Exposed surface area of cylinder

$= 2\pi(x)(y) + \pi x^2$

$= 2\pi x\left(\frac{100}{x^2} - \frac{16}{3}x\right) + \pi x^2$

$= \frac{200\pi}{x} - \frac{32}{3}\pi x^2 + \pi x^2$

$= \frac{200\pi}{x} - \frac{29}{3}\pi x^2.$

$\therefore$ Total surface area, $A = 11\pi x^2 + \frac{200\pi}{x} - \frac{29}{3}\pi x^2$

$= \frac{200\pi}{x} + \frac{4}{3}\pi x^2$

$= 20\pi\left(\frac{10}{x} + \frac{x^2}{15}\right)$ (shown).

9. (iii) $\frac{dA}{dx} = 20\pi\left(-\frac{10}{x^2} + \frac{2}{15}x\right)$

At stationary values, $\frac{dA}{dx} = 0$,

$20\pi\left(-\frac{10}{x^2} + \frac{2}{15}x\right) = 0.$

$\frac{10}{x^2} = \frac{2}{15}x.$

$x^3 = 75.$

$x = \sqrt[3]{75}.$

$\frac{d^2A}{dx^2} = 20\pi\left(\frac{20}{x^3} + \frac{2}{15}\right)$

When $x = \sqrt[3]{75}$,

$\frac{d^2A}{dx^2} = 20\pi\left(\frac{20}{75} + \frac{2}{15}\right)$

$= 8\pi > 0$ (min.)

$\therefore$ Stationary value of $x = \sqrt[3]{75}$ or 4.22 (3 s.f.)
and this yields a minimum value of A.

10. (i) $y = \frac{a}{x+b}$

$\frac{1}{y} = \frac{x+b}{a}$

$\frac{1}{y} = \frac{1}{a}x + \frac{b}{a}.$

Plot $\frac{1}{y}$ against x: Gradient of line $= \frac{1}{a}$
$\frac{1}{y}$ intercept $= \frac{b}{a}.$

x	0.5	1.0	2.5	3	4	
y	1.33	1.14	0.95	0.73	0.62	
$\frac{1}{y}$	0.75	0.88	1.05	1.37	1.61	(2d.p.)

$\therefore$ value of y incorrectly recorded is 0.95.

10. (ii) From graph, correct value of $\frac{1}{y} = 1.25 \Rightarrow y = \frac{1}{1.25} = 0.80$

$\therefore$ correct value of y when $x = 2.5$ is 0.80

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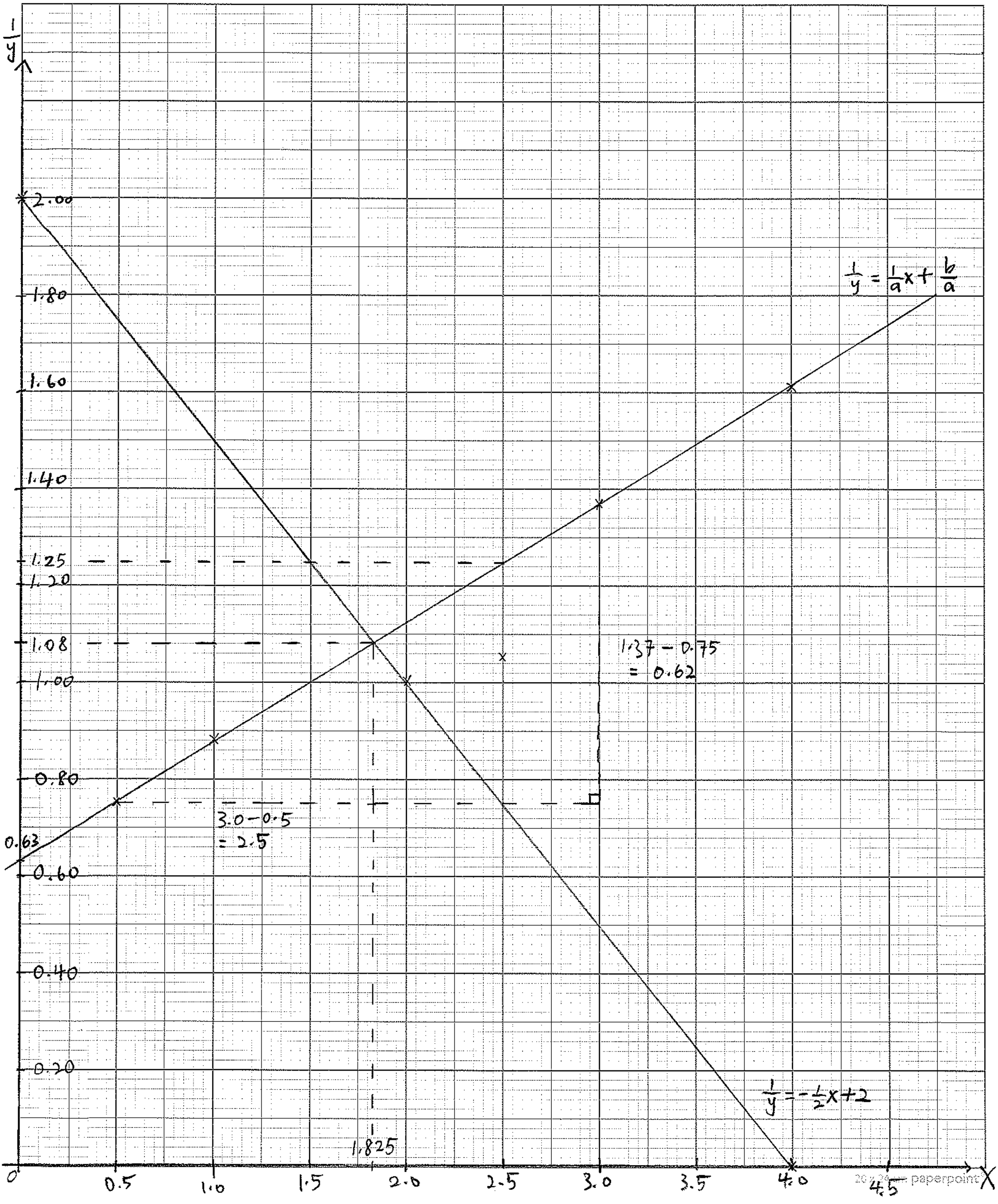
Subject : Q. 10(ii) Graph of $\frac{1}{y}$ against x. Class : __________ Date : __________



10. (iii) From graph, gradient of straight line $= \dfrac{1.37 - 0.75}{3.0 - 0.5}$

$= \dfrac{0.62}{2.5}$

$= 0.248$

$\Rightarrow \frac{1}{a} = 0.248.$

$\therefore a = \frac{1}{0.248}$

$= 4.03$ (3 s.f.)

From graph, $\frac{1}{y}$ intercept $= 0.63$

$\frac{b}{a} = 0.63$

$\frac{b}{4.032} = 0.63$

$\therefore b = 0.63 \times 4.032$

$= 2.54$ (3 s.f.)

10. (iv) $y = \frac{a}{x+b} \Rightarrow \frac{1}{y} = \frac{1}{a}x + \frac{b}{a}.$

$\frac{1}{y} + \frac{1}{2}x = 2 \Rightarrow \frac{1}{y} = -\frac{1}{2}x + 2.$

$\frac{1}{y} = -\frac{1}{2}x + 2.$

x	0	2	4
$\frac{1}{y}$	2	1	0

From graph, coordinates of intersection point between $\frac{1}{y} = \frac{1}{a}x + \frac{b}{a}$ and $\frac{1}{y} = -\frac{1}{2}x + 2$ is $(1.825, 1.08)$.

$\Rightarrow x = 1.825$

$\Rightarrow \frac{1}{y} = 1.08$

$y = \frac{1}{1.08}$

$= 0.926$ (3 s.f.)

$\therefore x = 1.825, \quad y = 0.926$

11. (i) $h = 1.1 \sin kt + 1.8$.

$-1 \leq \sin kt \leq 1$.

$-1.1 \leq 1.1 \sin kt \leq 1.1$

$-1.1 + 1.8 \leq 1.1 \sin kt + 1.8 \leq 1.1 + 1.8$.

$0.7 \leq h \leq 2.9$

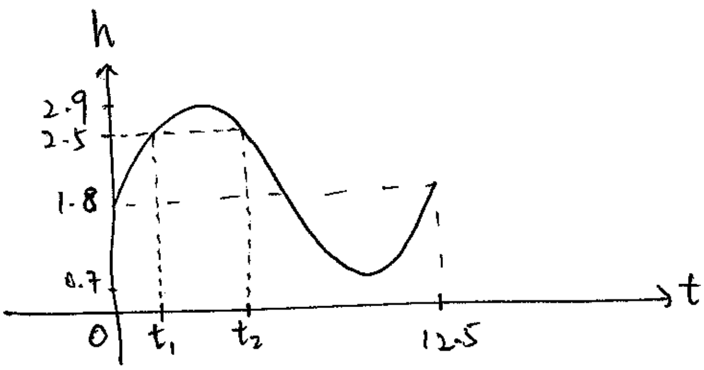
$\therefore$ Height of highest tide = 2.9m.

11. (ii) $\frac{2\pi}{k} = 12.5$

$k = \frac{2\pi}{12.5}$

$\therefore k = \frac{4\pi}{25}$. (shown)

11. (iii) $h = 1.1 \sin \frac{4\pi}{25}t + 1.8$.



From graph, when $h = 2.5$,

$1.1 \sin \frac{4\pi}{25}t + 1.8 = 2.5$

$\sin \frac{4\pi}{25}t = \frac{7}{11}$

basic $\angle \frac{4\pi}{25}t = 0.6898$ (4 s.f.)

$\frac{4\pi}{25}t = 0.6898, \pi - 0.6898$

$t = 1.372, 4.878$

$\Rightarrow t_1 = 1.372, t_2 = 4.878$.

$\therefore$ Duration of patrol duty shift

$= t_2 - t_1$

$= 4.878 - 1.372$

$= 3.506$ hours $\approx$ 3 hours 30 minutes.

12. (i) $\frac{d^2y}{dx^2} = 12x + p$.

$\frac{dy}{dx} = \int \frac{d^2y}{dx^2}\, dx$

$= \int 12x + p \, dx$

$= 6x^2 + px + c$ for some constants p and c.

When $x = 3$, $\frac{dy}{dx} = 0$.

$6(3)^2 + p(3) + c = 0$.

$3p + c = -54$. ①

When $x = 1$, $\frac{dy}{dx} = -4$.

$6(1)^2 + p(1) + c = -4$.

$p + c = -10$ ②

① → ②: $p - 54 - 3p = -10$.

$2p = -44$

$p = -22$

When $p = -22$, $c = 12$

$\frac{dy}{dx} = 6x^2 - 22x + 12$.

$y = \int \frac{dy}{dx}\, dx$

$= 2x^3 - 11x^2 + 12x + d$ for some constant d.

When $x = 1$, $y = 12$.

$2(1)^3 - 11(1)^2 + 12(1) + d = 12$.

$d = 9$.

$\therefore p = -22$

$\therefore$ equation of curve is $y = 2x^3 - 11x^2 + 12x + 9$.

12. (ii) $\frac{dy}{dx} = 6x^2 - 22x + 12$

$= 6(x^2 - \frac{11}{3}x) + 12$

$= 6\left[x^2 - \frac{11}{3}x + (\frac{11}{6})^2 - (\frac{11}{6})^2\right] + 12$

$= 6(x - \frac{11}{6})^2 - \frac{49}{6}$.

minimum point of gradient $\frac{dy}{dx}$ function is $(\frac{11}{6}, -\frac{49}{6})$.

$\therefore$ minimum gradient $= -\frac{49}{6}$, corresponding value of $x = \frac{11}{6}$.

12. (ii) (alternative)

Let G denote the gradient function $\frac{dy}{dx}$.

$\Rightarrow G = 6x^2 - 22x + 12$.

$$\frac{dG}{dx} = 12x - 22.$$

At stationary points, $\frac{dG}{dx} = 0$.

$$12x - 22 = 0.$$
$$x = \frac{22}{12}$$
$$= \frac{11}{6}.$$

$$\frac{d^2G}{dx^2} = 12 > 0 \text{ (min)}$$

When $x = \frac{11}{6}$, $G = 6\left(\frac{11}{6}\right)^2 - 22\left(\frac{11}{6}\right) + 12$

$$= -\frac{49}{6} \text{ (min)}.$$

$\therefore$ minimum gradient of the curve $= -\frac{49}{6}$

and corresponding value of $x = \frac{11}{6}$.