Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2017

SECONDARY 4

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **11 September 2017**

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/ 100

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 The equation of a curve is $y=\dfrac{e^{2x}}{3x}$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Given that x is changing at a constant rate of 0.05 units per second, find the rate of change of y when $x = 2$. [2]

(iii) Explain why the curve has one stationary point and state its x-coordinate. [2]

2 The roots of the quadratic equation $2x^2-3x+5=0$ are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

(i) Show that $\alpha+\beta=\dfrac{3}{5}$. [3]

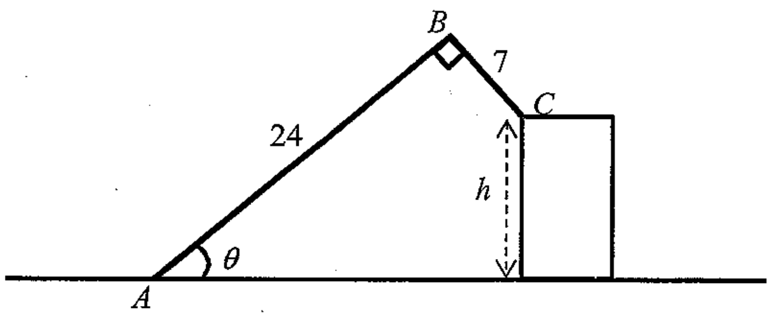
(ii) Find the value of $\alpha^3+\beta^3$. [2]

(iii) Find a quadratic equation whose roots are $\alpha^2+2\beta$ and $\beta^2+2\alpha$. [3]

3 **(i)** Find the constant term in the binomial expansion of $\left(x-\dfrac{1}{2x^3}\right)^8$. [3]

(ii) Given that the coefficient of $\dfrac{1}{x^4}$ in the expansion of $\left(1+\dfrac{4}{x^2}\right)^n+\left(x-\dfrac{1}{2x^3}\right)^8$ is 1049, find the value of n. [5]

4



The diagram shows a conveyor belt ABC to transport boxes from the horizontal ground to upright shelves at a height h m, where $AB = 24$ m, $BC = 7$ m and angle $ABC = 90°$. The part AB of the conveyor belt makes an acute angle θ with the horizontal ground and the angle θ can vary.

(i) Show that $h=p\sin\theta+q\cos\theta$, where p and q are constants to be found. [2]

(ii) Express h in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0°<\alpha<90°$. [4]

(iii) Find the value of θ for which the height of the shelves is 15 m. [2]

5 A particle leaves a fixed point O and moves in a straight line so that, t s after leaving O, its velocity, v m/s, is given by $v = t - 9 + \frac{16}{t+1}$. The particle comes to an instantaneous rest at point A and a little later at point B.

(i) Find the acceleration of the particle at A. [4]

(ii) Calculate the distance between A and B. [5]

(iii) Showing all your working, explain why the particle is again at O at some instant during the third second. [2]

6 It is given that $f(x) = 3x^3 + px^2 - 8x - 20$ leaves a remainder of -8 when divided by $x+1$.

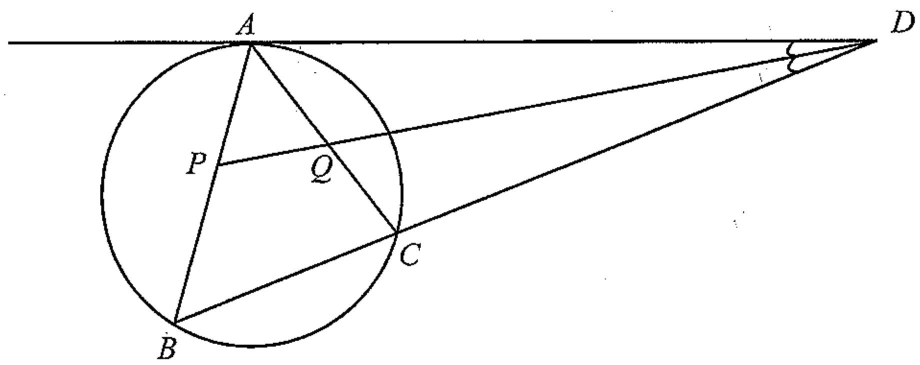
(i) Find the value of p. [2]

(ii) Find the range of values of x for which $f(x)$ is a decreasing function. [3]

(iii) By showing clearly your working, factorise $f(x)$. [3]

(iv) Hence, solve the equation $3(2^{3y}) + p(2^{2y}) - 8(2^y) - 20 = 0$. [3]

7



In the diagram, line AD is a tangent to the circle at A. The points B and C lie on the circle and BCD is a straight line. The line PQD bisects the angle ADC and intersects line AB at P and line AC at Q. Prove that

(i) triangle APQ is isosceles, [4]

(ii) $AP \times CD = AD \times CQ$. [3]

8 **(a)** Find the set of values of k for which the curve $y = x^2 + kx + 3k - 7$ lies completely above the line $y = 1$. [4]

(b) Express $3\log_5 x + \log_x 5 = 2$ as a quadratic equation and explain why there are no real solutions. [3]

(c) Solve the equation $\lg(x^2 + 8x) - \lg(x+1) = 2\lg 4$. [4]

9 **(i)** Show that $\dfrac{5\sin 2x + 2\cos 2x - 2}{1+\cos 2x} = 5\tan x - 2\tan^2 x$. [3]

(ii) Solve the equation $\dfrac{5\sin 2x + 2\cos 2x - 2}{1+\cos 2x} = \dfrac{3}{4}\tan x$ for $0° \le x \le 360°$. [4]

10 The equation of a circle, C_1, is $x^2 + y^2 + 8x + 6y = 11$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

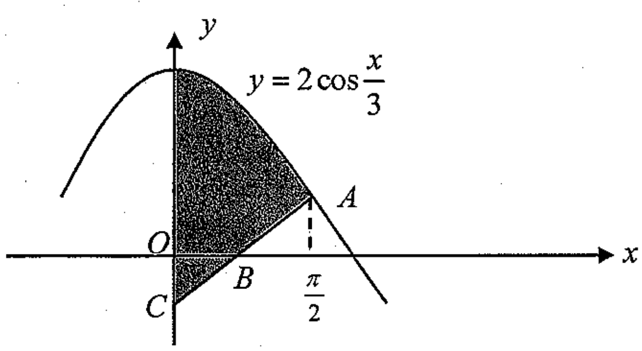
A second circle, C_2, passes through the points $P(1,\ 15)$ and $Q(-7,\ 7)$.
A line with gradient 2 passes through the centre of C_1 and C_2.

(ii) Show that the equation of the line is $y = 2x + 5$. [1]

(iii) Find the coordinates of the centre of C_2. [5]

(iv) Given that C_2 has radius a units, find the range of a such that $(1,\ 2)$ lies within C_2. [2]

11



The diagram shows part of the curve $y = 2\cos\dfrac{x}{3}$, where A lies on the curve. The x-coordinate of A is $\dfrac{\pi}{2}$ and the normal to the curve at A meets the x-axis at B and the y-axis at C.

(i) Find the coordinates of B and of C, leaving your answers in its exact form. [6]

(ii) Find the area of the shaded region. [6]

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