Candidate Name:	Class:	Index No:	Calculator Model:

DUNMAN SECONDARY SCHOOL

Where..... discernment, discipline, daring, determination & duty become a part of life.

END OF YEAR EXAMINATION 2017

SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS 4047

1040 – 1240 h 5 JULY 2017

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

For Examiner's Use

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten your work separately as follows and attach one Answer Cover Page on top for each set:
Questions 1 – 4
Questions 5 – 12

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2}ab \sin C.$$

1. It is given that $f(x) = ax^2 - 4(bx - 1)$ has equal roots. If one of the roots of $f(x)$ *is* $-\frac{1}{3}$, find the value of a and of b. [4]

2. Find the coordinates of the stationary point of the curve $y = \frac{x-1}{\sqrt{2x-3}}$ for $x > \frac{3}{2}$, and determine the nature of this stationary point. [6]

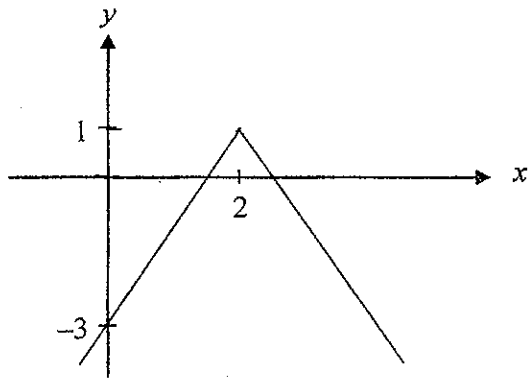
3. It is given that $\int_3^5 h(x-2)^2\,dx = 13$, where h is a constant.

   (i) Find the value of $\int_1^3 h(x-2)^2\,dx$. [4]

   (ii) Express $\int_3^5 \left[h(x-2)^2 + p\right]dx$ in terms of the constant p. [2]

4. A certain vessel initially contains water to a height of 25 cm. A small tap is opened at the bottom of the vessel and the water leaks away such that after t seconds, the rate of decrease of the height, h cm, of water in the vessel is given by $(2t - 10)$ cm s^{-1}. Calculate the time taken for the vessel to empty. [4]

5.



The diagram above shows part of the graph of $y = 1 - |2x - 4|$. In each of the following cases determine the number of intersections of the line $y = mx + c$, justifying your answer.

(i) $m = 2$ and $c > -3$, [2]

(ii) $m = -2$ and $c = 1$, [2]

(iii) $m = \frac{1}{2}$ and $c = 0$. [2]

6. (a) It is given that $2^{x-2} \times 3^{x+2} = 6^{2x}$. Find the exact value of 6^x. [3]

(b) Find the value of x such that $e^{3x+1} = 3^{2x-3}$. [4]

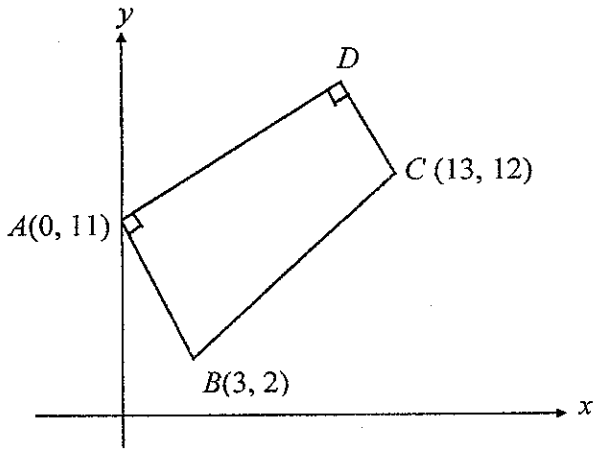
7. A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity is given by $v = 16(2e^{-2t} - e^{-t})$ m/s. Find

(i) the acceleration of the particle, [2]

(ii) the time when the particle reaches its greatest distance from O, [2]

(iii) its greatest distance from O. [2]

8.



$ABCD$ is a trapezium in which AB is parallel to DC and AD is perpendicular to both AB and DC. The coordinates of A, B and C are (0, 11), (3, 2) and (13, 12) respectively.

(i) Find the equations of DC and AD. [4]

(ii) Find the coordinates of the point D. [3]

(iii) Calculate the area of the trapezium $ABCD$. [2]

9. The roots of the quadratic equation $3x^2 + 2x - 8 = 0$ are α and β .

(i) Calculate the value of $\alpha^2 + \beta^2$. [4]

(ii) Find the quadratic equation with integer coefficients such that the roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [3]

10. (a) Given that x is measured in radians and $10 < x < 13$, find the value of x such that

$$3\operatorname{cosec}\left(\frac{2x+1}{3}\right) = 5.$$ [4]

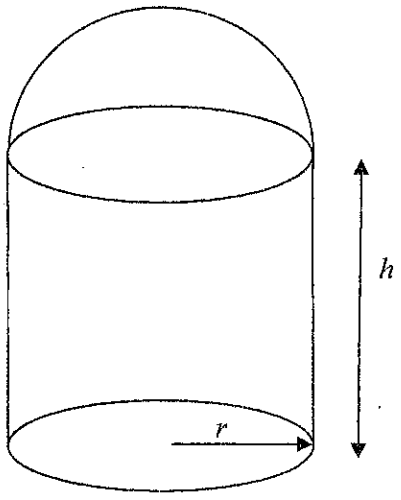
(b) Solve the equation $3\sec\theta\tan\theta + 1 + \tan^2\theta = 0$ for $0^\circ < \theta < 360^\circ$ [5]

11. Variables x and y are related by the equation $y = \frac{p}{x^2 - q}$, where p and q are constants. When the values of x^2y are plotted on the vertical axis against y on the horizontal axis, a straight line is obtained. Given that the intercept on the x^2y axis is 6 and that the gradient of the line is 7, find

(i) the values of p and q. [3]

(ii) the positive value of x when $xy = 1$. [3]

12.



[Volume of a sphere $= \frac{4}{3}\pi r^3$, surface area of a sphere $= 4\pi r^2$]

A can is in the shape of a closed cylinder with a hemisphere at one end. Its volume is 45π cm^3. Taking r cm as the radius of the cylinder and h cm as its height,

(i) express h in term of r, [3]

(ii) show that the external surface area A of the can is given by $A = \frac{5\pi r^2}{3} + \frac{90\pi}{r}$. [2]

(iii) hence find the value of r for which A is a minimum and the minimum value of A. [5]

End of Paper 1

ERRATUM

Please replace question 7 and 11 on the question paper with the following questions.

7. A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity is given by $v = 16(2e^{-2t} - e^{-t})$ m/s. Find

(i) the acceleration of the particle when $t = 2s$, [2]

(ii) the time when the particle reaches its greatest distance from O, [2]

(iii) its greatest distance from O. [2]

11. Variables x and y are related by the equation $y = \frac{p}{x^2 - q}$, where p and q are constants.

When the values of x^2y are plotted on the vertical axis against y on the horizontal axis, a straight line is obtained. Given that the line passes through (1, 13) and has gradient 5,

Find

(i) the values of p and q, [3]

(ii) the positive value of x when $xy = 1$. [3]