Candidate Name:	Class:	Index No:	Calculator Model:

DUNMAN SECONDARY SCHOOL

Where...... discernment, discipline, daring, determination & duty become a part of life.

PRELIMINARY EXAMINATION 2017

SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS 4047/02

0800 – 1030 h

6 JULY 2017

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

For Examiner's Use

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten your work separately as follows and attach one Answer Cover Page on top for each set:
Questions 1 – 4
Questions 5 – 11

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

This question paper consists of 7 printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2}ab \sin C.$$

Answer **all** the questions.

1 **(a)** Given that P is obtuse and $\tan P = -\frac{12}{5}$, without using a calculator, find the value of $\sin P$ and of $\cos P$. [2]

(b) Prove the identity $\tan 2A(2\cos A - \sec A) = 2\sin A$. [4]

2 It is given that $y = \frac{\cos x}{2-\sin x}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a + b\sin x}{(2-\sin x)^2}$,

(i) Find the value of each of the integers a and b. [5]

(ii) Using your answer to part **(i)**, evaluate $\int_0^{\frac{\pi}{2}} \frac{a + b\sin x}{(6-3\sin x)^2}\,\mathrm{d}x$. [3]

3 **(a)** **(i)** Find the first three terms in the expansion of $(1-2x)^5$ in ascending powers of x, simplifying the coefficients. [2]

(ii) Given that the first three terms in the expansion of $(a+bx)(1-2x)^5$ are $2 + cx + 10x^2$, state the value of a and hence find the value of b and of c. [3]

(b) The fourth term in the binomial expansion of $\left(px + \frac{q}{x}\right)^n$ is independent of x.

Find the value of n. [3]

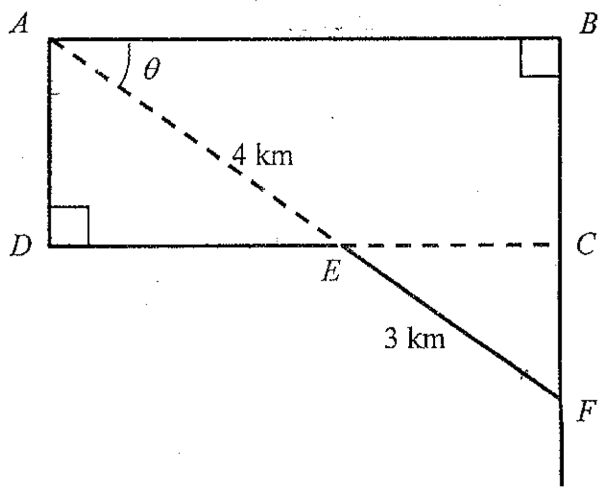
4 **(i)** Sketch the graph of $y = 7x^{\frac{1}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{7}x^{-\frac{3}{2}}$ for $x > 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

5



The diagram shows a Long Distance Race Route *FBADEF*.
ABCD is a rectangular plot of land.
A line through *A*, at an angle θ to *AB* intersects *DC* at *E* and *BC* produced at *F*, where $45° \le \theta < 90°$.
Runners for this race will start from point *F* and run along the straight paths *FB*, *BA*, *AD*, *DE* and return along the path *EF* to finish at *F*.
$AE = 4$ km and $EF = 3$ km.
The total length of the race is *L* km.

(i) Show that *L* can be expressed as $a\sin\theta + b\cos\theta + c$, where *a*, *b* and *c* are constants to be found. [3]

(ii) Express *L* in the form $R\sin(\theta + \alpha) + c$, where $R > 0$ and α is an acute angle. [3]

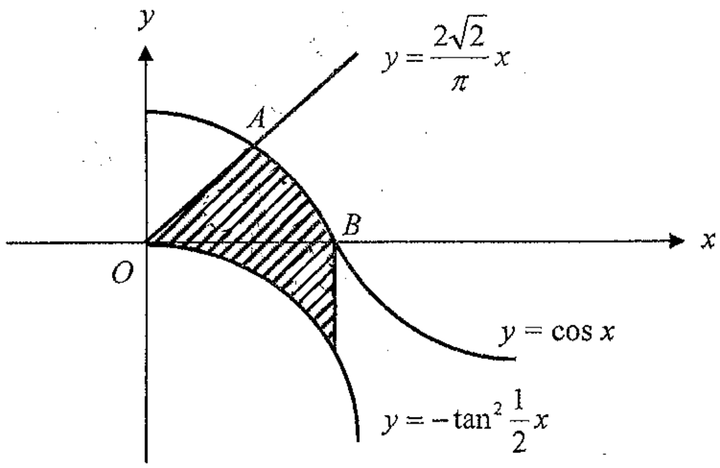
The total length of the race is found to be 18 km.

(iii) Find the value of θ. [2]

6 The diagram shows part of the graphs of $y = \cos x$, $y = -\tan^2 \frac{1}{2}x$ and $y = \frac{2\sqrt{2}}{\pi}x$.

The graphs of $y = \cos x$ and $y = \frac{2\sqrt{2}}{\pi}x$ intersects at the point A where the y-coordinate is $\frac{1}{\sqrt{2}}$.

The graph $y = \cos x$ cuts the x-axis at the point B.



(i) Calculate the coordinates of the points A and B. [2]

(ii) Hence find the area of the shaded region. [7]

7 Given that $\frac{ax^3 + x^2 + 6x + b}{2x^2 + x - 1} = 3x - 1 + \frac{cx + 4}{2x^2 + x - 1}$,

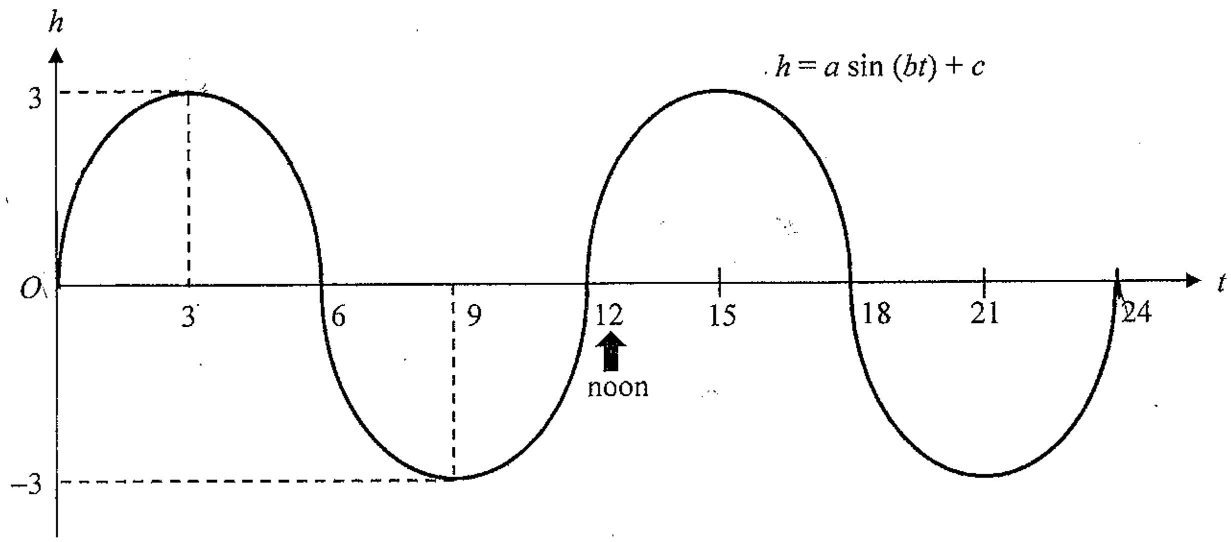
(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part **(i)**, find

(ii) $\int \frac{ax^3 + x^2 + 6x + b}{2x^2 + x - 1}\,dx$. [6]

8 The height, h metres, of the tide above mean sea level on 24 January at Cape Town is modelled approximately by $h = a \sin(bt) + c$, where t is the number of hours after midnight.

The graph for $h = a \sin(bt) + c$, $0 \leq t \leq 24$, is shown below.



(i) State the values of a, b and c. [3]

(ii) What is the maximum height of the tide above mean sea level? [1]

(iii) Find the height of the tide above mean sea level at 4 pm. [1]

(iv) A ship can cross the harbor when the tide is at least 2 m above mean sea level. When will crossing be possible on 24 January? [4]

9 A curve has the equation $y = f(x)$, where $f(x) = \dfrac{x+5}{3x+1}$ for $x > 0$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether f is an increasing or decreasing function. [1]

(iv) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

10 **(a)** The line $x-3y+1=0$ intersects the circle $(x-2)^2+(y-1)^2=10$ at the points A and B.

(i) Find the coordinates of A and of B. [5]

(ii) Determine if AB is a diameter of the circle. [2]

(b) Prove that the line $y=3x+8$ is tangent to the circle $x^2+y^2-4x-8y+10=0$ and find the point at which the line is tangent to the circle. [5]

11 **(a)** Solve $\log_x 81 = \log_3 x$. [4]

(b) A 40°F roast is cooked in a 350°F oven. After 2 hours, the temperature of the roast is 125°F. The temperature of the roast, T°F, as a function of its time in the oven, x hours, follows Newton's Law of Heating:

$$T = T_a + (T_0 - T_a)e^{-kx},$$

where T = temperature of the roast after x hours,
T_a = the surrounding temperature and in this case, it is 350°F,
T_0 = the initial temperature of the roast and in this case, it is 40°F.

(i) Find the value of k and show that the temperature of the roast, T, can be represented with the formula

$$T = 350 - 310e^{-0.1602x}.$$ [3]

(ii) The roast is cooked completely when the temperature, T, reaches 165°F. Find the amount of time taken, in hours and minutes, for the roast to be cooked completely. [2]

(c) Given that $\log_x(p^2q^3)=m$ and $\log_x(pq^2)=n$, find $\log_x \sqrt{pq}$ in terms of m and n. [3]

End of Paper