Name: ______________________________ (   )   Class: __________

MONTFORT SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2017

Secondary 4 Express/ 5 Normal Academic

ADDITIONAL MATHEMATICS
Paper 1

4047/01
02 Aug 2017 (Wed)

10.05 am

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Setters: Mdm R Tang, Ms Norazidah, Ms C Choy

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The angles x and y are acute angles measured in degrees.

(i) If $\cos 2x$ is negative, state the range of values of x for which x lies in. [1]

(ii) If $\sin 4y$ is negative and $\cos 4y$ is positive, what is the range of values of y for which y lies in? [1]

(iii) If the conditions in (i) and (ii) remain true, determine with explanations whether the following are positive or negative:

(a) $\tan(x+y)$, [1]

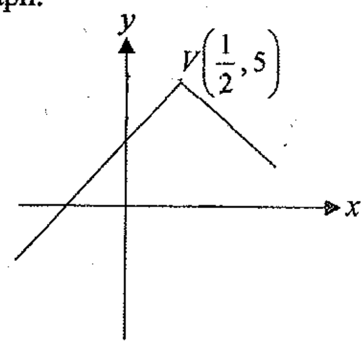
(b) $\tan 3y$. [1]

2 (i) On the same axes, sketch the curves $y = 8x^{\frac{3}{2}}$ and $y = 6x^{\frac{1}{2}}$. [2]

(ii) Find the coordinates of the point(s) at which the curves in part (i) intersect. [3]

(iii) Hence find the coordinates of the point(s) at which the curves $y = -8x^{\frac{3}{2}}$ and $y = -6x^{\frac{1}{2}}$ intersect. [1]

3 The figure shows part of the graph of $y = h - |kx - 1|$, where $V\left(\frac{1}{2}, 5\right)$ is the vertex of the graph.



(i) Find the value of h and of k. [2]

(ii) Determine the set of values of m for which the line $y = mx + 5$ intersects the graph $y = h - |kx - 1|$ at

(a) one point, [2]

(b) two distinct points. [1]

4 (i) Prove that $\dfrac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 1 + \dfrac{1}{2}\sin 2x$. [3]

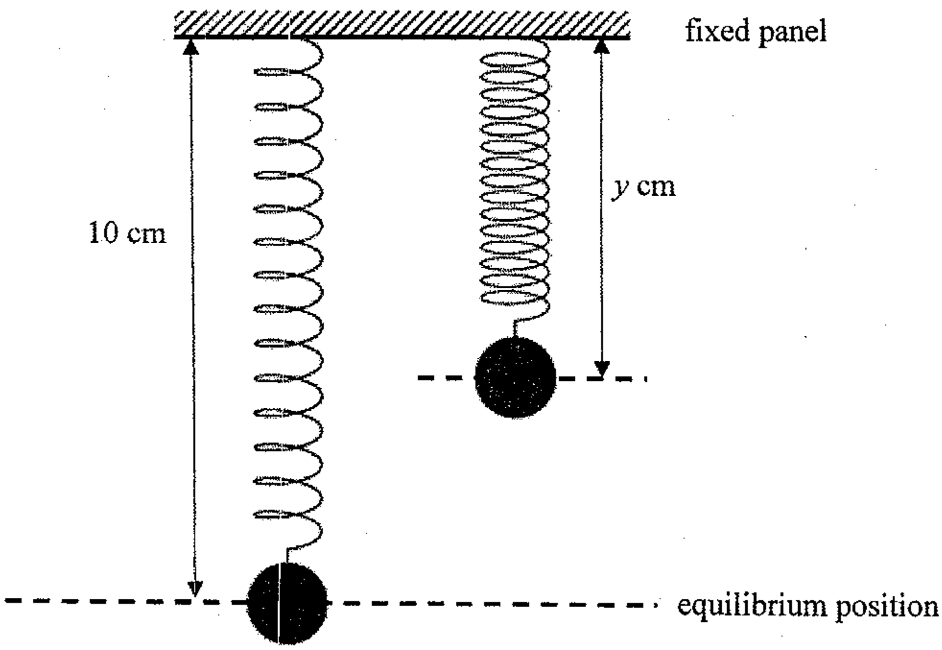
(ii) Hence sketch the graph of $y = \dfrac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$ for $-\pi \le x \le \pi$. [2]

5 (i) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\dfrac{1}{4}x^2(2\ln x - 1)\right] = x\ln x$. [3]

(ii) Hence, find the exact value of $\int_1^4 \dfrac{x\ln x}{6}\,\mathrm{d}x$. [3]

6 The diagram shows a mass suspended from the end of a spring. The distance of the spring from the fixed panel is 10 cm at its equilibrium position. The spring is then being compressed by 5 cm and released. The distance, y cm, of the spring from the fixed panel t seconds after being released can be modelled by the equation

$$y = -5\cos\left(\frac{2\pi t}{3}\right) + 10.$$



(i) Find the maximum distance of the spring from the fixed panel. [1]

(ii) Find the time taken by the spring to reach its equilibrium position for the first time after it is being released. [2]

(iii) Explain why the distance of the spring from the fixed panel may not be represented by the equation $y = -5\cos\left(\dfrac{2\pi t}{3}\right) + 10$ in real life. [2]

7 The sides AB and BC of a triangle are $(2\sqrt{6}+\sqrt{3})$ cm and $\left(7\sqrt{2}-3\right)$ cm respectively and angle $ABC = 120°$.

(i) Show that the area of triangle ABC is $\left(\dfrac{3\sqrt{2}+75}{4}\right)\text{cm}^2$. [2]

(ii) Hence, find the perpendicular distance from A to BC, leaving your answer in the form of $(a+b\sqrt{2})$ cm, where a and b are rational numbers. [4]

8 (a) Find the values of m for which the line $y=3x+m$ is a tangent to the curve $y=\dfrac{3}{1-x}$. [3]

(b) Show that the roots of the equation $px^2+(2p+q)x+2q=0$ are real for all values of p and q. Hence, write down a relation between p and q if the roots are real and distinct. [4]

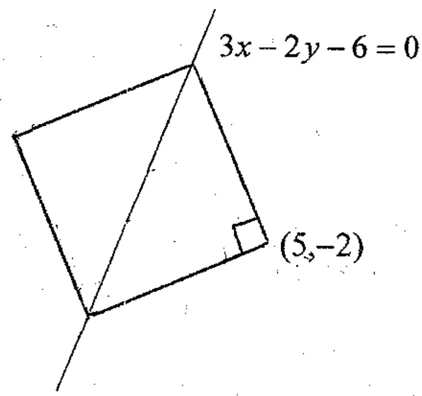
9 (i) Express $\dfrac{3x^2+23x-45}{(x+3)^2}$ in partial fractions. [4]

(ii) Hence find $\displaystyle\int\frac{3x^2+25x-39}{(x+3)^2}\,dx$. [4]

10 The diagram shows a square with a vertex at the point $(5,-2)$ and a diagonal along the straight line $3x-2y-6=0$.

(i) Find the equation of the second diagonal of the square. [2]

(ii) Find the coordinates of the other vertices. [6]



11 The function f is defined for all real values of x by

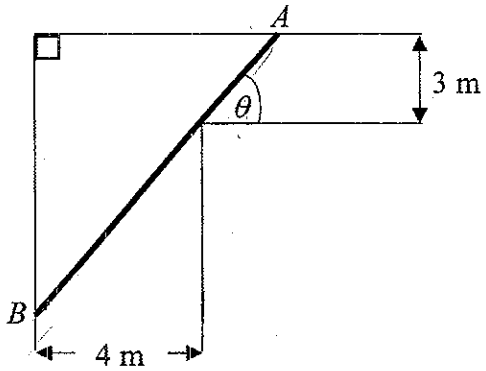
$$f(x) = (\cos x - \sin x)(17\cos x - 7\sin x).$$

(i) Show that $f(x)$ may be expressed in the form $R\cos(2x+\alpha)+k$, where $R>0$, $0<\alpha<\frac{\pi}{2}$ and k is a constant. [6]

(ii) Determine the greatest and least values of $\frac{78}{f(x)+14}$. [3]

(iii) Find the principal value of x for which the greatest value in part **(ii)** occurs. [2]

12 The diagram shows two corridors in a particular floor of a new building that meet at right angles and are 3 m and 4 m wide respectively. AB is a thin metal tube which must be kept horizontal and cannot be bent as it moves around from one corridor to the other. θ, measured in degrees, is the acute angle marked on the given diagram.



(i) Show that the length AB is given by $L = \frac{4}{\cos\theta} + \frac{3}{\sin\theta}$. [2]

(ii) Find an expression for $\frac{dL}{d\theta}$. [2]

(iii) Find θ when $\frac{dL}{d\theta} = 0$. [3]

(iv) You are the manager of this particular building. What is your advice to contractors who will be carrying out renovation jobs in the new building? [2]

End of paper