Name: ______________________________ (   )   Class: __________

MONTFORT SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2017

Secondary 4 Express/ 5 Normal Academic

ADDITIONAL MATHEMATICS
Paper 2

4047/02
03 Aug 2017 (Thu)

10.00 am

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of 6 printed pages and 0 blank page.

Setters: Ms Norazidah, Ms R Tang, Ms C Choy

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the equation $\log_2 \sqrt{5x+1} - \log_4 64 + 2\log_9 3 = \log_4 (x-2)$. [5]

2 The roots of a quadratic equation $ax^2 + bx + 6 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Given that $\alpha + \beta = \frac{1}{2}$ and $\alpha\beta = 4$, find the value of a and of b. [3]

(ii) Find the quadratic equation $mx^2 + nx + p = 0$ whose roots are α^3 and β^3 and m, n and p are integers. [5]

3 A curve has the equation $y = 2(5-x)(3x+1)^{-1}$, where $x \neq -\frac{1}{3}$.

(i) Show that y is a decreasing function. [3]

(ii) Find the value(s) of y at which y is decreasing at 8 times the rate of increase of x. [5]

4 (i) Given that $u = 3^x$, express $7 - 3^{2x+1} - 2(3^{-x} + 3^x)$ as an expression in u. [2]

(ii) Hence, by showing your working clearly, solve $7 - 3^{2x+1} - 2(3^{-x} + 3^x) = 0$. [6]

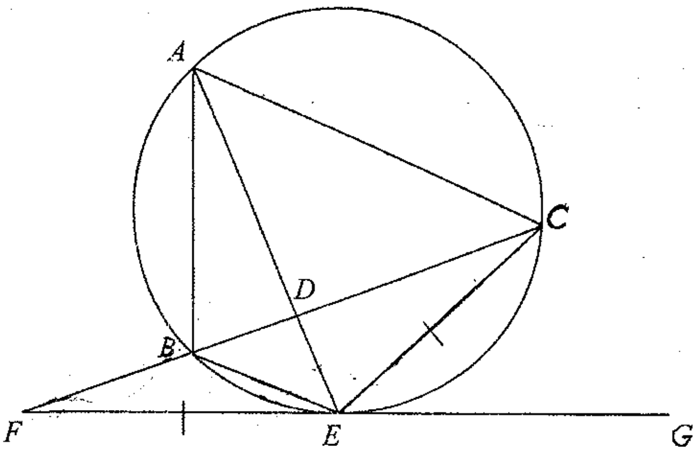
5 (i) Find the term independent of x in the expansion of $\left(-2x^2 - \frac{1}{2x}\right)^9$. [4]

(ii) Hence, or otherwise, find the term independent of x in

$$\left(\frac{5}{4} - 4x^3\right)\left(-2x^2 - \frac{1}{2x}\right)^9.$$

[4]

6 The diagram shows triangle ABC is inscribed on the circle. FEG is a tangent to the circle at E such that $EF = EC$. FBC is a straight line and EA cuts BC at D.



(i) Prove that triangles BEF and ECF are similar. [3]

(ii) Prove that angle $CAD = 2 \times$ angle BAD. [4]

(iii) Prove that $(FE)^2 = FB \times FC$. Hence, prove that $CE \times FE = FB \times FC$. [3]

7 **(i)** Given that $(\cos 2x + \cos x)^2 + (\sin 2x + \sin x)^2 = A + B\cos x$, find the value of A and of B. [4]

(ii) Hence, or otherwise, solve the equation

$$(\cos 2x + \cos x)^2 + (\sin 2x + \sin x)^2 + 1 = \frac{2}{\operatorname{cosec}^2 x} - \cos x$$

for $-180° \leq x \leq 180°$. [5]

8 In a chemical reaction, a solid substance slowly dissolves in a liquid. At the start of the reaction, there are P_0 kilograms of the substance. After t minutes, there are P kilograms of the substance to be dissolved. It is known that P and t are related by the equation $P = P_0 e^{-kt}$, where k is a constant. The table below shows measured values of P and t.

t (minutes)	10	20	30	40	50
P (kg)	245	155	88	55	33

(i) Using suitable variables, draw, on graph paper, a straight line and hence estimate the value of P_0 and of k. [6]

(ii) Estimate the time needed for 20% of the solid substance to be dissolved. [2]

(iii) State the value that P approaches if this research is to continue for a long period of time. Justify your answer. [2]

9 The equation of a circle, C_1, with centre A, is $x^2 + y^2 - 18x - 4y - 15 = 0$.

(i) Find the coordinates of A and the radius of C_1. [3]

A straight line with a negative gradient cuts the x-axis at an angle θ with $\tan\theta = \frac{3}{4}$ and passes through $B(3, -6)$.

(ii) Show that the straight line is a tangent to C_1. [2]

The highest point on the circle is D.

(iii) Write down the equation of the tangent to the circle at D. [1]

(iv) Find the coordinates of the point at which the tangents to the circle at B and D intersect. [3]

A second circle, C_2 passes through A, D and E, where angle DEA is $90°$.

(v) Find the equation of C_2. [3]

10 A particle travelling in a straight line passes through a fixed point O with a velocity of 4.8 m/s. The acceleration, a m/s^2, of the particle, t s after passing through O, is given by $a = -5e^{-t}$. The particle comes to instantaneous rest at the point P.

(i) Show that the time at which the particle reaches P is $-\ln 0.04$. [6]

(ii) Find the distance of the particle from O at this instant. [4]

11 **(i)** The curve has the equation $y = (3x^2 - 3x + 1)(x - 1)$.

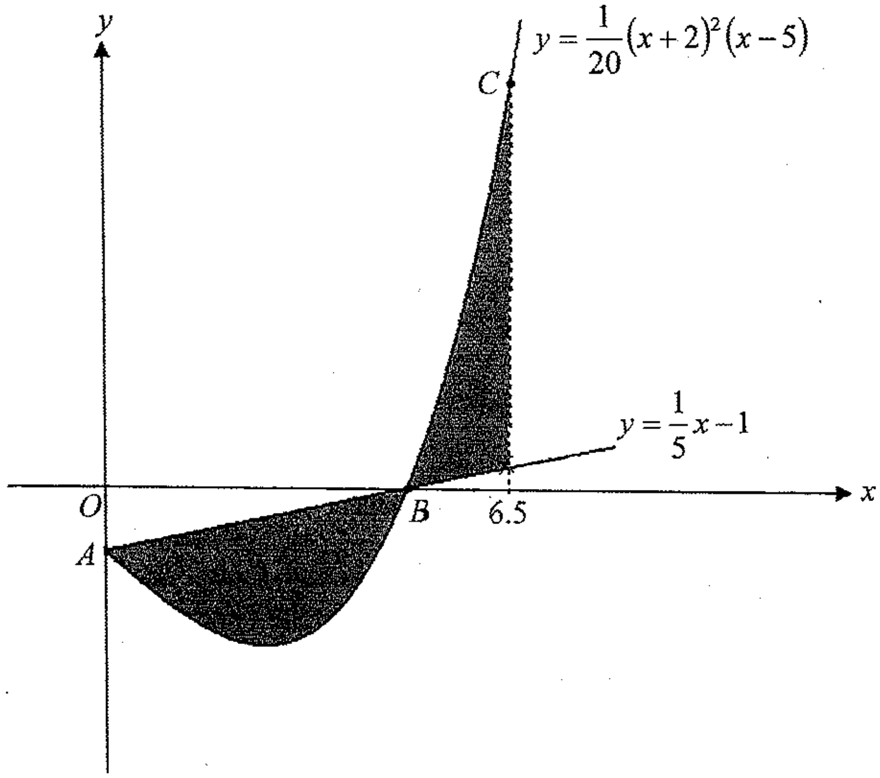
(a) Show that the gradient of the tangent to the curve can never be negative. [2]

(b) Show that the tangent to the curve at $x = \frac{2}{3}$ is parallel to x-axis. [1]

(c) Determine the nature of the point of the curve at $x = \frac{2}{3}$. [2]

(d) Show that the normal ~~to the tangent~~ to the curve at $x = 2$ cuts the y-axis at $7\frac{1}{8}$. [2]

(ii) The diagram shows part of the curve $y=\frac{1}{20}(x+2)^2(x-5)$ and the line $y=\frac{1}{5}x-1$.
The curve and the line intersect at points A and B at $x=0$ and 5 respectively.
Given that the x-coordinate of C is 6.5, find the total area of the shaded regions. [5]



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