Name: ______________________________ ( )	Class : Sec ______

TAMPINES SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2017

SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4047/01**
04 Aug 2017 **2 hours**

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

1. Write your name, class and index number in the spaces at the top of this page and on **all the work you hand in**.
2. Write in **dark blue** or **black** pen on both sides of the paper.
3. You may use an HB pencil for any diagrams or graphs.
4. Do not use staples, paper clips, highlighters, glue or correction fluid.

5. Answer **all** the questions.
6. Write your answers on the separate Answer Paper provided.
7. Give non-exact numerical answers correct to **3 significant figures**, or **1 decimal place** in the case of angles in degrees, unless a different level of accuracy is specified in the question.
8. The use of an **approved** electronic calculator is expected, where appropriate.
9. You are reminded of the need for clear presentation in your answers.

10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [ ] at the end of each question or part question.
12. The total number of marks for this paper is **80**.

This document consists of **6** printed pages

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a) (i)** Write down and simplify the first three terms in the expansion of $(1-2x)^6$ in ascending powers of x. [3]

(ii) Hence, or otherwise, obtain the coefficient of x^2 in the expansion of $(5+4x-3x^2)(1-2x)^6$. [2]

(b) (i) Write down the general term in the binomial expansion of $\left(x-\frac{p}{x^2}\right)^7$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, given that the coefficient of x is 336 in the expansion of $\left(x-\frac{p}{x^2}\right)^7$, determine the positive value of p. [3]

2 Find the range of values of k for which the line $y=k(x-1)$ does not intersect the curve $y=x^2+6x+k$. [5]

3 Solve the equation $3^{1-3x} \div \left(27^{\frac{-x}{3}} \times 9^{x+1}\right) = 243$. [4]

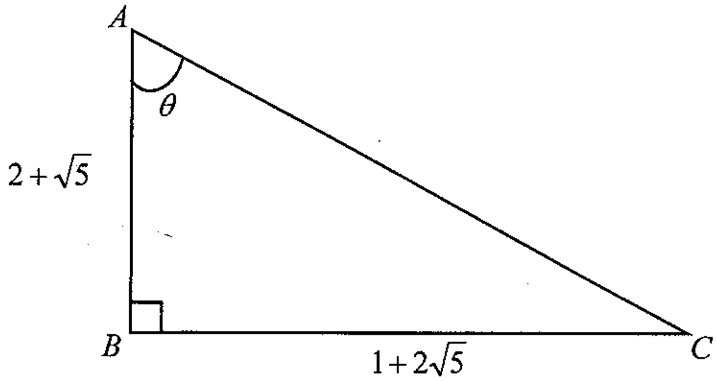
4 **(i)** Prove that $\dfrac{\operatorname{cosec}\theta}{\cot\theta+\tan\theta} = \cos\theta$. [3]

(ii) Hence solve $\dfrac{\operatorname{cosec}x}{\cot x+\tan x} = 0.5$ for $0 \le x \le \pi$ radians, giving your answers in terms of π. [3]

5 **(i)** Differentiate $\sin x\cos x$ with respect to x, giving your answer in terms of $\sin x$. [3]

(ii) Find the value of $\displaystyle\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx$ [3]

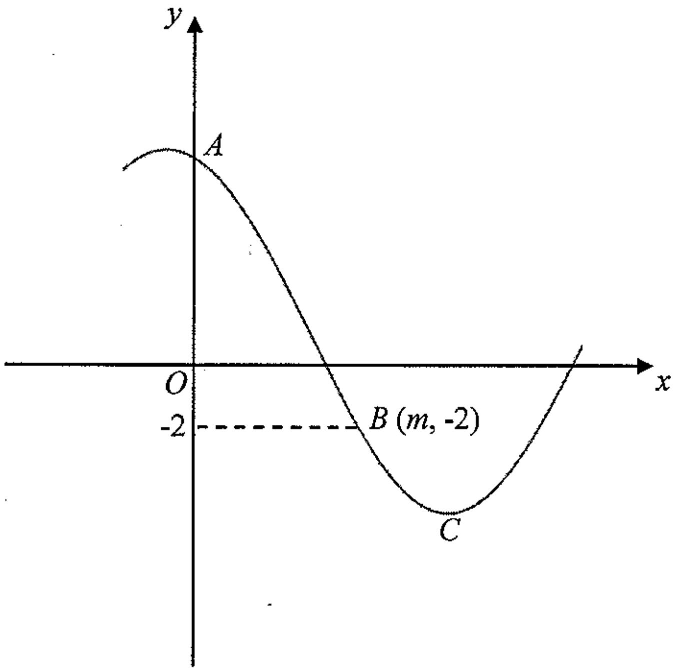
6 The diagram shows a right angled triangle *ABC*. The side *AB* is $(2+\sqrt{5})$ cm and the side *BC* is $(1+2\sqrt{5})$. Angle *BAC* is θ.



(i) Express $\tan\theta$ in the form $a+b\sqrt{5}$, where a and b are integers. [3]

(ii) Hence find $\sec^2\theta$ in the form $c+d\sqrt{5}$, where c and d are integers. [3]

7 The diagram shows part of the curve $y=4\cos x+5\sin x$ for $0\le x\le 2\pi$. The points *A*, *B* and *C* are on the curve such that *A* lies on the *y*-axis, the coordinates of *B* are $(m, -2)$ and *y* is a minimum at *C*.



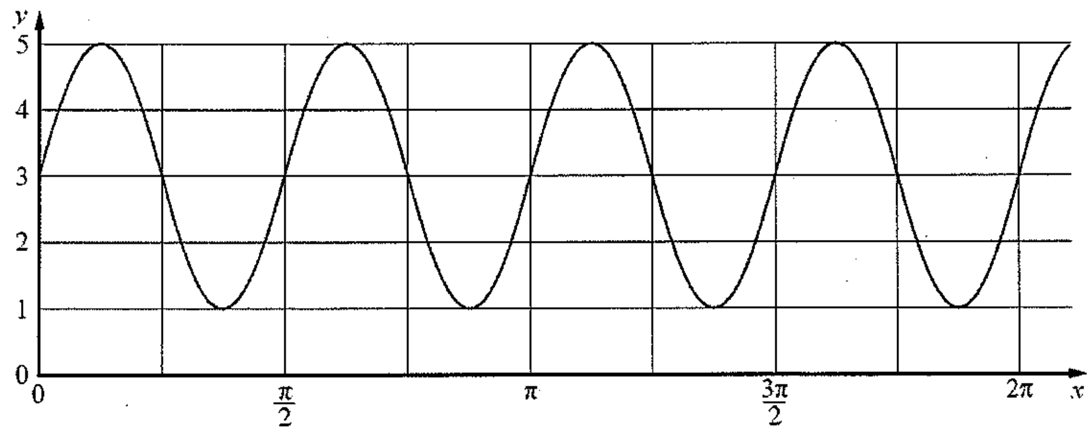
(i) Express $4\cos x+5\sin x$ in the form $R\cos(x-\alpha)$, where $R>0$ and α is acute. [3]

(ii) Find the coordinates of *A* and *C*. [3]

(iii) Calculate the value of *m*. [3]

8 A curve that shows the relationship between two variables x and y, passes through the point $Q(-1, 3)$. The curve has a gradient of 2 at Q. Given that $\frac{d^2y}{dx^2} = -5$, find the equation of the curve. [5]

9 **(a)** The diagram shows part of the graph of $y = a + b\sin(4c)x$.
Find the value of each of the constants a, b and c. [3]



(b) It is given that θ is the principal value of $\tan^{-1}\sqrt{3}$ and ϕ is the principal value of $\tan^{-1}(-1)$.

(i) Express θ in terms of π. [1]

(ii) Express ϕ in terms of π. [1]

10 A function is given by $f(x) = \left(3 - \frac{1}{2}x\right)^3$. P is a point on the graph of $y = f(x)$, such that its coordinates is $(2, k)$ where k is a constant.

(i) Show that $f(x)$ is a decreasing function for all values of x. [2]

(ii) Find the equation of the tangent at P. [3]

11 The table below shows the experimental values of two variables x and y.

x	2	4	6	8
y	9.6	38.4	105	232

It is known that x and y are related by the equation $y = ax^3 + bx$, where a and b are constants.

(i) A straight line graph is plotted to represent this information with $\frac{y}{x}$ on the vertical axis. State the variable which must be plotted on the horizontal axis. [1]

(ii) Plot the straight line. [2]

(iii) Use your graph to estimate the value of a and of b. [3]

(iv) Estimate the value of x for which $2y = 25x$. [2]

12 Two particles, A and B, leaves a point O at the same time and travel along the same straight line. Particle A starts from rest and travels with a uniform velocity of 3 m/s.
The velocity of particle B is given by $v = 3t^2 - 20t + 24$ where t is the time in seconds after leaving O.

(i) Find the acceleration of the particle B when $t = 5$ seconds. [2]

(ii) By showing clearly your working, explain why the particle B changes its direction of motion twice. [2]

(iii) Find an expression for the displacement of the particle A and of particle B from O in terms of t. [4]

(iv) Calculate the distance from O at which particle A first collides with particle B. [3]

End of Paper

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1(a)(i)	$(1-2x)^6 \approx 1+\binom{6}{1}(-2x)+\binom{6}{2}(-2x)^2$ $=1-12x+60x^2$
(ii)	$(5+4x-3x^2)(1-2x)^6$ $\approx (5+4x-3x^2)(1-12x+60x^2)$ Coeff of $x^2 = 5(60)+4(-12)+(-3)(1)=249$
(b)(i)	$\binom{7}{r}x^{7-r}\left(\frac{-p}{x^2}\right)^r$
(ii)	Power of $x = 7-3r$
(iii)	$7-3r=1$ $r=2$ Coeff of $x=\binom{7}{2}(-p)^2=336$ $p=4$
2	$kx-k=x^2+6x+k$ $x^2+(6-k)x+2k=0$ $(6-k)^2-4(1)(2k)<0$ $k^2-20k+36<0$ $(k-18)(k-2)<0$ $2<k<18$
3	$3^{1-3x}\div(3^{-x}\times 3^{2x+2})=3^5$ $3^{1-3x}\div(3^{x+2})=3^5$ $1-3x-(x+2)=5$ $x=-\frac{3}{2}$ or [Accept $x=-1.5$]
4(i)	LHS: $\frac{\operatorname{cosec}\theta}{\cot\theta+\tan\theta}=\frac{1/\sin\theta}{\cos\theta/\sin\theta+\sin\theta/\cos\theta}$ $=\frac{1/\sin\theta}{(\cos^2\theta+\sin^2\theta)/\sin\theta\cos\theta}$ $=\frac{1}{\sin\theta}\div\frac{1}{\sin\theta\cos\theta}$ $=\cos\theta$ = RHS (Proven) Alternative Method $\frac{\operatorname{cosec}\theta}{\cot\theta+\tan\theta}=\frac{1/\sin\theta}{1/\tan\theta+\tan\theta}$

	$= \dfrac{1/\sin\theta}{(1+\tan^2\theta)/\tan\theta}$ $= \dfrac{1}{\sin\theta} \div \dfrac{\sec^2\theta}{\tan\theta}$ $= \dfrac{1}{\sin\theta} \times \sin\theta\cos\theta$ $= \cos\theta$ = RHS (Proven)
(ii)	$\cos x = 0.5$ $\alpha = \dfrac{\pi}{3}$ $x = \dfrac{\pi}{3}$ or $x = \dfrac{5\pi}{3}$
5(i)	$\dfrac{d}{dx}(\sin x\cos x) = \sin x(-\sin x) + \cos x(\cos x)$ $= 1 - 2\sin^2 x$
(ii)	$\dfrac{d}{dx}(\sin x\cos x) = 1 - 2\sin^2 x$ $[\sin x\cos x]_{-\pi/4}^{\pi/4} = \int_{-\pi/4}^{\pi/4} 1dx - \int_{-\pi/4}^{\pi/4} 2\sin^2 x dx$ $[\sin x\cos x]_{-\pi/4}^{\pi/4} = [x]_{-\pi/4}^{\pi/4} - \int_{-\pi/4}^{\pi/4} 2\sin^2 x dx$ $\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx = 0.285$
6(i)	$\tan\theta = \dfrac{1+2\sqrt{5}}{2+\sqrt{5}}$ $= \dfrac{1+2\sqrt{5}}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$ $= 8 - 3\sqrt{5}$
(ii)	$\sec^2\theta = 1 + (8-3\sqrt{5})^2$ $= 1 + 64 - 48\sqrt{5} + 45$ $= 110 - 48\sqrt{5}$
7(i)	$R = \sqrt{41}$ $\alpha = \tan^{-1}\left(\dfrac{5}{4}\right)$ $= 0.896$ $\sqrt{41}\cos(x - 0.896)$

(ii)	$A=(0,4)$ $\cos(x-0.89605)=-1$ $x-0.89605=\pi$ $x=4.04$ $C=(4.04,-6.40)$
(iii)	$\cos(m-0.89605)=-0.3123$ $m-0.89605=1.888$ $m=2.78$
8	$\frac{dy}{dx}=-5x+c$ $-5(-1)+c=2$ $c=-3$ $\frac{dy}{dx}=-5x-3$ $y=-\frac{5}{2}x^2-3x+c$ $3=-\frac{5}{2}(-1)^2-3(-1)+c$ $c=\frac{5}{2}$ $\therefore y=-\frac{5}{2}x^2-3x+\frac{5}{2}$
9(a)	$a=3$ $b=2$ $c=1$
(b) (i) (ii)	$\theta=\frac{\pi}{3}$ $\phi=-\frac{\pi}{4}$
10 (i)	$f'(x)=3\left(3-\frac{1}{2}x\right)^2\left(-\frac{1}{2}\right)$ $f'(x)=-\frac{3}{2}\left(3-\frac{1}{2}x\right)^2$ < 0 for all values of x $\therefore f(x)$ is a decreasing function for all values of *x*.
(ii)	$f'(2)=3\left(3-\frac{1}{2}(2)\right)^2\left(-\frac{1}{2}\right)$ = -6 $y=f(2)=8=k$ $8=-6(2)+c$ $\Rightarrow c=20$ $\therefore y=-6x+20$
11(i)	x^2
(ii)	Refer to the graph

(iii)	$b=3$ $a=$ gradient of the graph $= \dfrac{\text{difference in vertical dist}}{\text{difference in horizontal dist}}$ $=0.411$
(iv)	$x^2 = 23.6$ $x = 4.86$
12(i)	$a = 6t - 20$ $a = 6(5) - 20 = 10ms^{-2}$
(ii)	Set $v = 3t^2 - 20t + 24$ equals to zero $\Rightarrow 3t^2 - 20t + 24 = 0$ $t = 1.57$ or $t = 5.10$ Since there are two possible values of t for which V = 0, hence there are two turning points for the particle B when it changes its direction
(iii)	Displacement of particle A = $3t$ Displacement of particle B = $\int 3t^2 - 20t + 24\,dt$ $= t^3 - 10t^2 + 24t + c$ At $t = 0$, $S = 0 \Rightarrow c = 0$ $S_B = t^3 - 10t^2 + 24t$
(iv)	$S_B = S_A$ $t^3 - 10t^2 + 24t = 3t$ $t^3 - 10t^2 + 21t = 0$ $t(t^2 - 10t + 21) = 0$ $t(t-3)(t-7) = 0$ Time when particle A first collides with particle B is 3 seconds Distance from O = $3 \times 3 = 9m$