| Name:______________________________( ) | Class : Sec ______ |
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# TAMPINES SECONDARY SCHOOL

## PRELIMINARY EXAMINATION 2017

*SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC*

**ADDITIONAL MATHEMATICS** **4047/02**
**28 Aug 2017** **2 hours 30 minutes**

Additional Materials: Answer Paper

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**READ THESE INSTRUCTIONS FIRST**

1. Write your name, class and index number in the spaces at the top of this page and on **all the work you hand in**.
2. Write in **dark blue** or **black** pen on both sides of the paper.
3. You may use an HB pencil for any diagrams or graphs.
4. Do not use staples, paper clips, highlighters, glue or correction fluid.

5. Answer **all** the questions.
6. Write your answers on the separate Answer Paper provided.
7. Give non-exact numerical answers correct to **3 significant figures**, or **1 decimal place** in the case of angles in degrees, unless a different level of accuracy is specified in the question.
8. The use of an **approved** electronic calculator is expected, where appropriate.
9. You are reminded of the need for clear presentation in your answers.

10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [ ] at the end of each question or part question.
12. The total number of marks for this paper is **100**.

This document consists of **8** printed pages

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta$ ABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Solve $\lg(5x-3)=0$. [2]

**(b)** Express $2\log_3 15-(\log_a 5)(\log_3 a)$, where $a>1$, as a single logarithm to base 3. [4]

2 The roots of the quadratic equation $3x^2-5x+2=0$ are $\alpha$ and $\beta$ where $\alpha>\beta$.

**(i)** Show that the value of $\alpha^2+\beta^2=\frac{13}{9}$. [3]

**(ii)** Find the value of $\alpha^3-\beta^3$. [4]

**(iii)** Find a quadratic equation with integer coefficients whose roots are $\alpha^2-\beta$ and $\alpha+\beta^2$. [4]

3 **(a)** Express $\frac{9x-11}{4-x^2}$ in partial fractions. [4]

**(b)** The term containing the highest power of the polynomial $f(x)$ is $3x^4$.
Two of the roots of the equation $f(x)=0$ are 1 and $k$.
Given that the quadratic factor of $f(x)$ is $x^2-3x+5$ and the remainder when $f(x)$ is divided by $x-2$ is 36,

**(i)** write down an expression in factorized form for $f(x)$ in terms of $k$, [3]

**(ii)** calculate the value of $k$, [2]

**(iii)** find the number of real roots of the equation $f(x)=0$, justifying your answer. [2]

4 **(a) (i)** Show that the equation $\operatorname{cosec}2x=8-2\cot^2 2x$ may be written in the form

$$2\operatorname{cosec}^2 2x+\operatorname{cosec}2x-10=0.$$ [1]

**(ii)** Find all values of $x$ between $0^0$ and $180^0$ for which $\operatorname{cosec}2x=8-2\cot^2 2x$. [7]

**(b)** The graph of $y=\left|(3x-2)^2-5\right|$ passes through the point $(c, 4)$.

**(i)** Find the possible values of the constant $c$. [4]

**(ii)** Sketch the graph of $y=\left|(3x-2)^2-5\right|$, labelling the coordinates of the turning point and of the points where the graph meets the axes. [4]

**5 (a)** The equation of a parabola is $y^2 = 8x$.

**(i)** Show that the point (2, -4) lies on the parabola. [1]

**(ii)** Sketch the graph of the parabola for $0 \le x \le 8$. [2]

**(b)** The equation of a circle is $x^2 + y^2 - 4x + 2y = 8$.

**(i)** State the coordinates of the centre of the circle. [1]

**(ii)** Calculate the area of the circle. [2]

**(iii)** By showing working clearly, explain whether the point (5, 2) lies inside the circle, outside the circle or on the circle. [2]

**6** Oil is dropped onto the centre of a flat circular surface of radius 24 cm. The oil spreads evenly and forms a circular patch growing at a rate of 3 $cm^2/s$.

When the circular patch has a radius of 15 cm, find in terms of $\pi$,

**(i)** the time taken to form the circular patch, [2]

**(ii)** the rate of increase of the radius of the oil patch at this instant, [3]

**7** The volume of a conical tent, $V$, is given by $V = \frac{1}{3}\pi\left(2h^2 + 4h - \frac{3}{4}h^3\right)$ where $h$ is the height.

**(i)** Given that $h$ varies, find the value of $h$ which gives a stationary value of $V$, giving your answer correct to 2 decimal places. [4]

**(ii)** Use the second derivative test to determine if $V$ is maximum or minimum. [3]

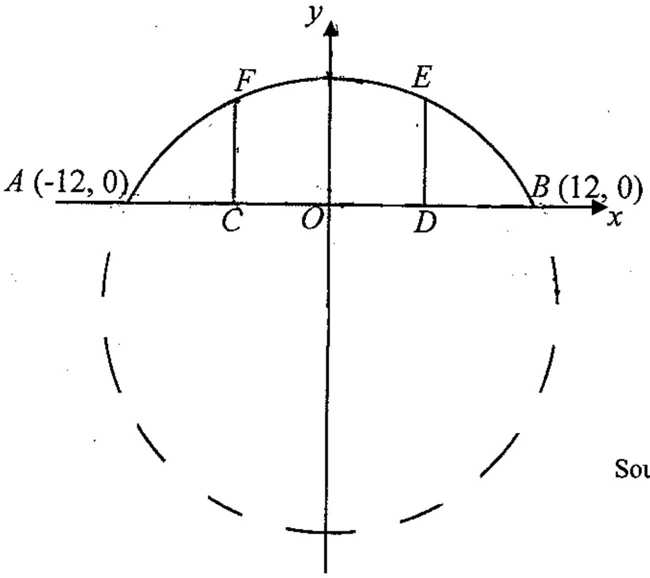
8 The diagram shows a simplified model (not drawn to scale) of the arch $AFTEB$ of an Elgin Bridge located near the Singapore River.

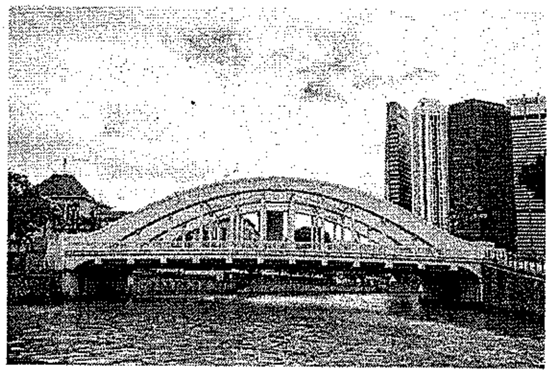
The bridge forms an arc of a circle and the length $AB$ forms a chord of the circle.

$AB$ is 24 m and the top of the bridge $T$ is 10 m vertically above $AB$.

$C$ and $D$ are the midpoints of $OA$ and $OB$ respectively.

$CF$ and $DE$ are two vertical pillars supporting the arch.



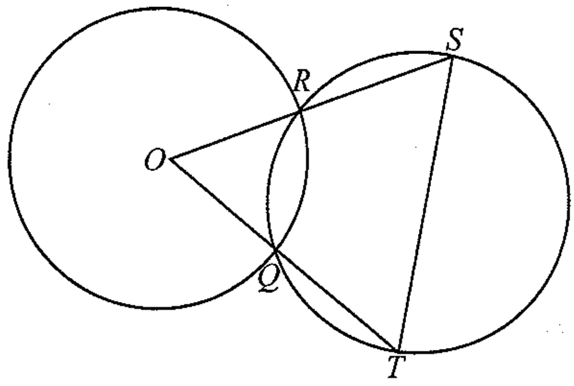



Source : https://en.wikipedia.org/wiki/Elgin_Bridge_(Singapore)

**(i)** Find the equation of the circle. [4]

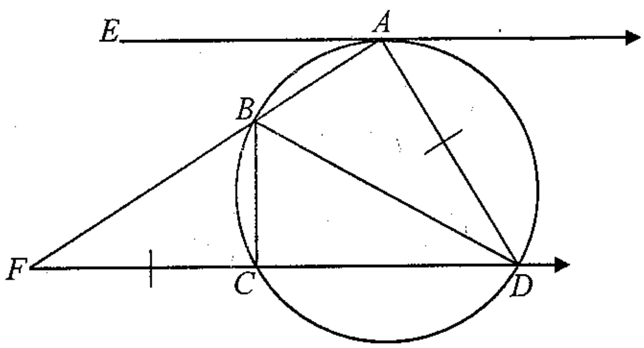
**(ii)** Calculate the height of the pillar $CF$. [2]

**9 (a)**




The above diagram shows two circles $C_1$ and $C_2$ with the same radius intersecting at $R$ and $Q$, which are the mid-points of $OS$ and $OT$ respectively. Given that $ST$ is a diameter of $C_2$, prove that triangle $ORQ$ is an equilateral triangle. [3]

**(b)**

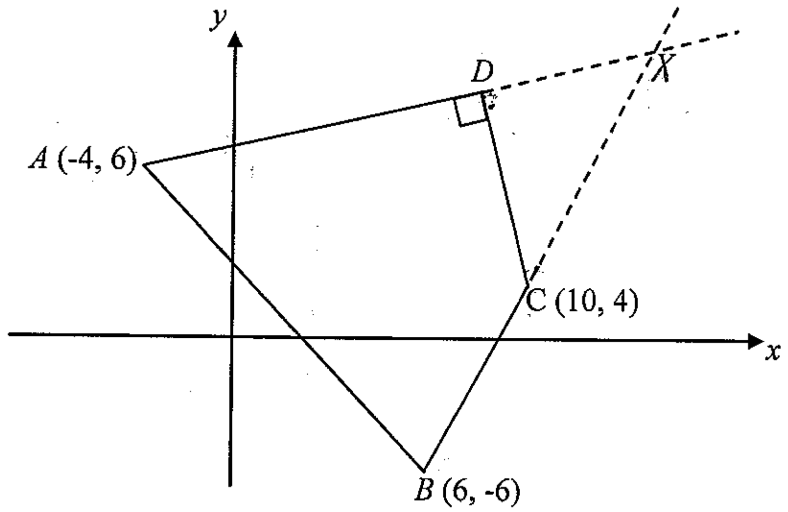



In the diagram above, $EA$ is a tangent to the circle at $A$. $FBA$ and $FCD$ are straight lines. $EA$ is parallel to $FD$ and $FC = AD$. By stating reasons clearly, prove that

**(i)** $\angle BAD = \angle BCF$, [1]

**(ii)** triangle $ABD$ is congruent to triangle $CBF$. [4]

**10 Solutions to this question by accurate drawing will not be accepted.**



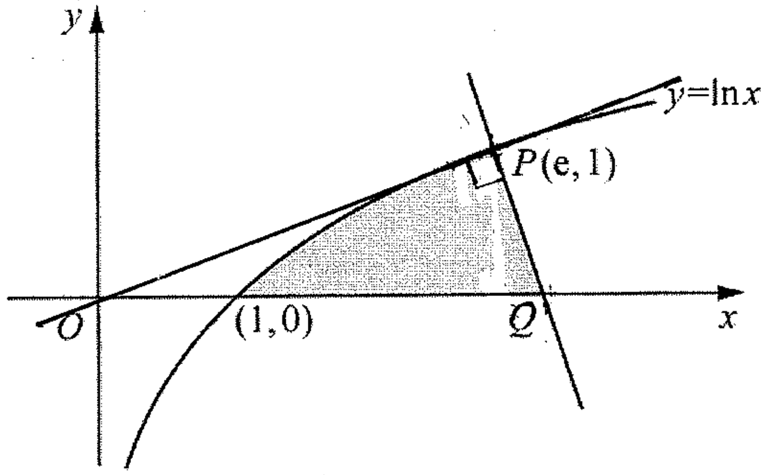

The diagram shows a quadrilateral $ABCD$ with vertices $A(-4, 6)$, $B\ (6, -6)$, $C(\ 10, 4)$ and $D$.
The angle $ADC = 90°$. The lines $BC$ and $AD$ are extended to intersect at point $X$.
Given that $C$ is the midpoint of $BX$,

**(i)** find the coordinates of $X$, [2]

**(ii)** calculate the coordinates of $D$, [6]

**(iii)** calculate the area of quadrilateral $ABCD$. [2]

**11**




The diagram shows part of the curve $y = \ln x$ intersecting the $x$-axis at the point (1, 0). The normal to the curve at point $P$ (e, 1) meets the $x$-axis at point $Q$.

**(i)** Show that the tangent to the curve at $P$ passes through the origin. [2]

**(ii)** Show that the coordinates of $Q$ is $\left(e + \dfrac{1}{e}, 0\right)$. [3]

**(iii)** Differentiate $x \ln x$ with respect to $x$. [2]

**Hence**, find

**(iv)** $\int \ln x dx$ and [2]

**(v)** the area of the shaded region. [3]

***End of Paper***

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| 1(a) | $5x-3=1$<br>$x=\frac{4}{5}$ [accept 0.8] | (b) | $\log_3 15^2-\frac{\log_3 5}{\log_3 a}\times\log_3 a$<br>$=\log_3\left(\frac{225}{5}\right)$<br>$=\log_3 45$ [accept $2+\log_3 5$] |
| 2(i) | $\alpha+\beta=\frac{5}{3}$<br>$\alpha\beta=\frac{2}{3}$<br>$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$<br>$=\frac{25}{9}-\frac{4}{3}=\frac{13}{9}$ | | |
| (ii) | $(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$<br>$=\frac{13}{9}-\frac{4}{3}$<br>$=\frac{1}{9}$<br>$\alpha-\beta=\sqrt{\frac{1}{9}}=\frac{1}{3}$<br>$\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$<br>$=\frac{1}{3}\left(\frac{13}{9}+\frac{2}{3}\right)$<br>$=\frac{19}{27}$ [Accept 0.704] | (iii) | Sum of roots : $\alpha^2-\beta+\alpha+\beta^2$<br>$=\alpha^2+\beta^2+\alpha-\beta$<br>$=\frac{16}{9}$<br>Pdt of roots : $(\alpha^2-\beta)(\alpha+\beta^2)$<br>$=\alpha^3+(\alpha\beta)^2-\alpha\beta-\beta^3$<br>$=\alpha^3-\beta^3+(\alpha\beta)^2-\alpha\beta$<br>$=\frac{13}{27}$<br>Quad eqn : $x^2-\frac{16}{9}x+\frac{13}{27}=0$<br>$27x^2-48x+13=0$ |
| 3(a) | $\frac{9x-11}{(2-x)(2+x)}=\frac{A}{2-x}+\frac{B}{2+x}$<br>$9x-11=A(2+x)+B(2-x)$<br>Using suitable substitutions of x to find the value of $A$ and $B$ such that<br>$A=\frac{7}{4}$ , $B=\frac{-29}{4}$<br>$\therefore\frac{9x-11}{(2-x)(2+x)}=\frac{7}{4(2-x)}-\frac{29}{4(2+x)}$ [Do not accept<br>$\frac{9x-11}{(2-x)(2+x)}=\frac{7/4}{(2-x)}-\frac{29/4}{(2+x)}$ | | |
| (b)(i) | $f(x)=3(x-1)(x-k)(x^2-3x+5)$ | | |
| (ii) | $f(2)=36$<br>$\Rightarrow 3(2-1)(2-k)(4-6+5)=36$<br>$k=-2$ | | |
| (iii) | For $x^2-3x+5=0$<br>Discriminant = $(-3)^2-4(1)(5)=-11<0$<br>$\therefore$ Two real roots for $f(x)=0$ | | |
| 4(a) (i) | $\operatorname{cosec}2x=8-2(\operatorname{cosec}^2 2x-1)$<br>$\operatorname{cosec}2x=8-2\operatorname{cosec}^2 2x+2$ | | |

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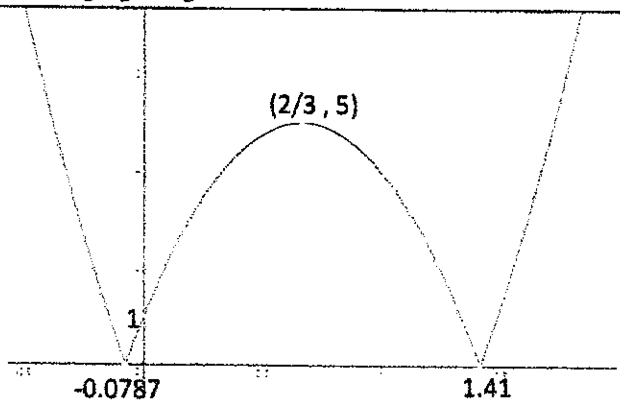
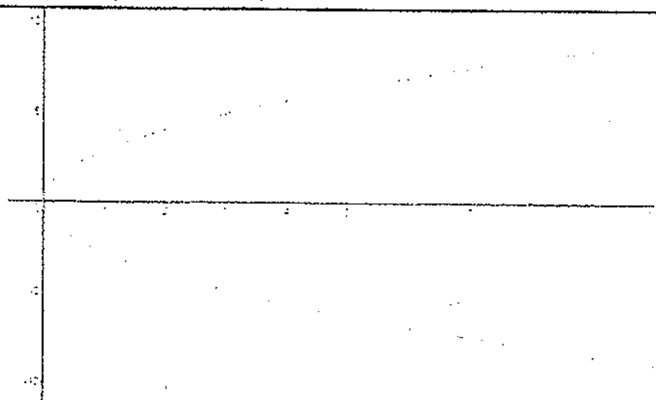
| | $2\cos ec^2 2x + \cos ec2x - 10 = 0$ |
| **(ii)** | $\cos ec2x = 8 - 2\cot^2 2x$<br>$\Rightarrow 2\cos ec^2 2x + \cos ec2x - 10 = 0$<br>$(2\cos ec2x + 5)(\cos ec2x - 2) = 0$<br>$\cos ec2x = -\frac{5}{2}$ or $\cos ec2x = 2$<br>$\sin 2x = -\frac{2}{5}$ or $\sin 2x = \frac{1}{2}$<br>$\alpha = 23.58^0$ $\alpha = 30^0$<br>$2x = 203.58^0, 336.42^0$ or $2x = 30^0, 150^0$<br>$x = 15^0,\ 75^0, 101.8^0, 168.2^0$ |
| **4(b)**<br>**(i)** | $4 = \lvert (3c-2)^2 - 5 \rvert$<br>$(3c-2)^2 - 5 = 4$ or $-4$<br>$c = -\frac{1}{3}, \frac{1}{3}, 1, \frac{5}{3}$ |
| **(ii)** |  |
| **5(a)**<br>**(i)** | When $x = 2$, $y^2 = 16$<br>$\Rightarrow y = \pm 4$<br>$\therefore (2, -4)$ lies on the parabola |
| **(ii)** |  |
| **(b)(i)** | (2 , -1) |
| **(ii)** | Radius of the circle = $\sqrt{2^2 + (-1)^2 - (-8)} = \sqrt{13}$<br>Area of the circle = $13\pi = 40.8$ sq units |
| **(iii)** | Let P be the point (5, 2)<br>Length of OP = $\sqrt{(5-2)^2 + (2-(-1))^2} = \sqrt{18}$<br>Since $\sqrt{18} > \sqrt{13}$ (radius of circle), hence P lies outside the circle. |
| **6** | $A = \pi 15^2$ |

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| (i) | Time taken to form the circular patch<br>$= \dfrac{\pi 15^2}{3} = 75\pi$ |
| (ii) | $\dfrac{dA}{dr} = 2\pi r$<br>$\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt}$<br>$3 = 2\pi(15) \times \dfrac{dr}{dt}$<br>$\dfrac{dr}{dt} = \dfrac{1}{10\pi}$ [accept $\dfrac{0.1}{\pi}$] |
| 7<br>(i) | $\dfrac{dV}{dh} = \dfrac{4}{3}\pi h + \dfrac{4}{3}\pi - \dfrac{3}{4}\pi h^2$<br>Set $\dfrac{dV}{dh} = 0 \Rightarrow \dfrac{4}{3}\pi h + \dfrac{4}{3}\pi - \dfrac{3}{4}\pi h^2 = 0$<br>$9h^2 - 16h - 16 = 0$<br>$h = \dfrac{16 \pm \sqrt{832}}{18}$<br>$h = 2.49$ reject $h = -0.71$ |
| (ii) | $\dfrac{d^2V}{dh^2} = \dfrac{4}{3}\pi - \dfrac{3}{2}\pi h$<br>When $h = 2.49$, $\dfrac{d^2V}{dh^2} = -7.54 < 0$<br>$\therefore V$ is maximum when $h = 2.49$ |
| 8(i) | Let the centre of the circle be ( p , q)<br>$10 - q = \sqrt{(12-p)^2 + (0-y)^2}$<br>Sub $p = 0 \Rightarrow 10 - q = \sqrt{144 + q^2}$<br>$q = -2.2$<br>Eqn of the circle is $x^2 + (y+2.2)^2 = 148.84$ |
| (ii) | Sub $x = 6$ into the eqn found in (i)<br>$6^2 + (y+2.2)^2 = 148.84$<br>$y = 8.42$ metres |
| 9(a) | Since R & Q are the midpoints of OS & OT respectively, hence by the midpoint theorem, $RQ$ is parallel to $ST$ and $ST = 2RQ$<br>$\therefore OR = OQ = RQ$ since $C_1$ and $C_2$ have the same radius, so triangle *ORQ* is an equilateral triangle |
| (b)<br>(i) | $\angle BCF = 180^0 - \angle BCD$ (supplementary angles)<br>$= \angle BAD$ (angles in opp segment) |
| (ii) | $\angle EAF = \angle BFC$ (alt angles)<br>$\angle EAF = \angle BDA$ (alt segment theorem)<br>$\therefore \angle BFC = \angle BDA$<br>Given that *FC* = *AD* and $\angle BAD = \angle BCF$ from (i)<br>$\therefore \Delta ABD \equiv \Delta CBF$ (ASA) |
| 10(i) | Let *X* be (a, b)<br>$\left(\dfrac{a+6}{2}, \dfrac{b-6}{2}\right) = (10, 4)$<br>$\therefore a = 14$ , $b = 14$ |

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| | $X = (14, 14)$ |
| **(ii)** | Gradient of AX = $\frac{4}{9}$<br>Eqn of line AX is $y = \frac{4}{9}x + \frac{70}{9}$ ------- (1)<br>Gradient of line DC = $-\frac{9}{4}$<br>Eqn of line DC is $y = \frac{9}{4}x + \frac{53}{2}$ ------ (2)<br>Solving the simultaneous eqns (1) and (2),<br>$x = \frac{674}{97}$ , $y = \frac{1054}{97}$<br>$\therefore D = \left(\frac{674}{97}, \frac{1054}{97}\right)$ |
| **(iii)** | Area of quadrilateral *ABCD*<br>$= \frac{1}{2}\begin{vmatrix} -4 & 6 & 10 & 674/97 & -4 \\ 6 & -6 & 4 & 1054/97 & 6 \end{vmatrix}$<br>$= \frac{1}{2}\left(24 + 24 + 108\frac{64}{97} + 41\frac{67}{97} - (36 - 60 + 27\frac{77}{97} - 43\frac{45}{97}\right)$ = 119 sq units |
| **11(i)** | $y = \ln x$<br>$\frac{dy}{dx} = \frac{1}{x}$<br>At P, $\frac{dy}{dx} = \frac{1}{e}$<br>So the eqn of the tangent is $y = \frac{1}{e}x + c$<br>Sub *P* (e, 1), $1 = \frac{1}{e}(e) + c \Rightarrow c = 0$<br>$\therefore$ The tangent to the curve at *P* passes through the origin |
| **(ii)** | Gradient of the normal at P = $-e$<br>Eqn of the normal at P is $y = -ex + 1 + e^2$<br>When y = 0, $0 = -ex + 1 + e^2$<br>$\Rightarrow x = e + \frac{1}{e}$<br>$\therefore Q = \left(e + \frac{1}{e}, 0\right)$ |
| **(iii)** | $\frac{d}{dx} x\ln x = 1 + \ln x$ |
| **(iv)** | $\int \ln x dx$ = $x\ln x - x + c$ |
| **(v)** | Area of the shaded region<br>$= \int_1^e \ln x dx + \frac{1}{2} \times \frac{1}{e} \times 1$<br>= 1.18 sq units |