MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4047/01

Paper 1

October/November 2018

2 hours

Additional Materials: Answer Paper
Graph Paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **3** blank pages.

Singapore Examinations and Assessment Board

CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $\sqrt{125^x} = \frac{5^{1-x}}{25}$, find the value of $\sqrt{125^x}$. [4]

2 A, B and C are the angles of a triangle.

(i) Show that $\tan C = -\tan(A + B)$. [2]

Angle $A = 45°$ and angle $B = 60°$.

(ii) **Without using a calculator**, find $\tan C$ in the form $a + \sqrt{b}$, where a and b are integers. [3]

3 Express $\frac{7x^2 - 12x + 17}{(2x - 1)(x^2 + 4)}$ in partial fractions. [6]

4 (a) Given that $3 + 2\sqrt{5}$ is a root of the equation $x^2 + ax + b = 0$, where a and b are integers, find the value of a and of b. [3]

(b) A rectangle of length $(6 + \sqrt{12})$cm has an area of $(24 + \sqrt{48})\text{cm}^2$. **Without using a calculator,** find the breadth of the rectangle, in cm, in the form $(c + d\sqrt{3})$, where c and d are integers. [3]

5 (i) On the same diagram sketch the curves $y^2 = 16x$ and $y = 6x^{-\frac{1}{2}}$. [3]

(ii) Find the coordinates of the point of intersection of the two curves. [2]

6 (i) Show that $\log_3 x + \log_9 x = \frac{3\lg x}{2\lg 3}$. [3]

(ii) Hence solve the equation $\log_3 x + \log_9 x = 4$. [2]

7 Water is poured, at a constant rate of $18\pi\,\text{cm}^3/\text{s}$, into a hemispherical bowl of radius 12 cm. When the depth of water directly below the centre is x cm, the volume, $V\,\text{cm}^3$, of water in the bowl is given by

$$V = \frac{1}{3}\pi x^2(36 - x).$$

Find

(i) the time taken for the depth of water directly below the centre to reach 9 cm, [3]

(ii) the rate of change of the depth of water directly below the centre at this time. [4]

8 (i) Show that $8\sin^2 x + 2\cos^2 x$ can be written as $a + b\cos 2x$, where a and b are integers. [2]

Hence

(ii) state the period and amplitude of $8\sin^2 x + 2\cos^2 x$, [2]

(iii) sketch the graph of $y = 8\sin^2 x + 2\cos^2 x$ for $0 \leqslant x \leqslant 2\pi$ radians. [3]

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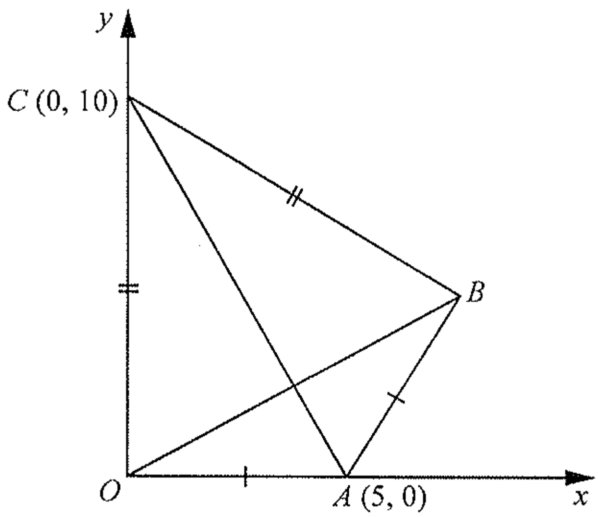

The diagram shows a kite $OABC$ in which $OA = AB$ and $OC = BC$. Given that the coordinates of A and C are (5, 0) and (0, 10) respectively, find

(i) the equation of OB, [3]

(ii) the coordinates of B. [4]

10 A gardener uses 20 m of fencing with which to enclose 2 flower beds. One bed is to be an equilateral triangle of side x m. The other bed is to be a circle of radius r m.

(i) Express r in terms of x. [1]

(ii) Show that the total area, $A\,\text{m}^2$, of the two flower beds is given by

$$A = \frac{\sqrt{3}\pi x^2 + (20 - 3x)^2}{4\pi}.$$

[3]

(iii) Given that x can vary, find the value of x which gives a stationary value of A. [4]

(iv) Find the nature of this stationary value and explain why the gardener might be disappointed. [2]

11 It is given that $f(x)$ is defined for $x > \frac{3}{2}$ and is such that $f'(x) = \frac{10x - 9}{2x - 3}$.

(i) Express $f'(x)$ in the form $a + \frac{b}{2x - 3}$, where a and b are constants. [2]

(ii) Hence explain whether $f(x)$ is an increasing or decreasing function. [1]

(iii) By considering $f''(x)$, explain whether $f'(x)$ is an increasing or decreasing function. [4]

(iv) Given that $f(2) = 8$, obtain an expression for $f(x)$. [4]

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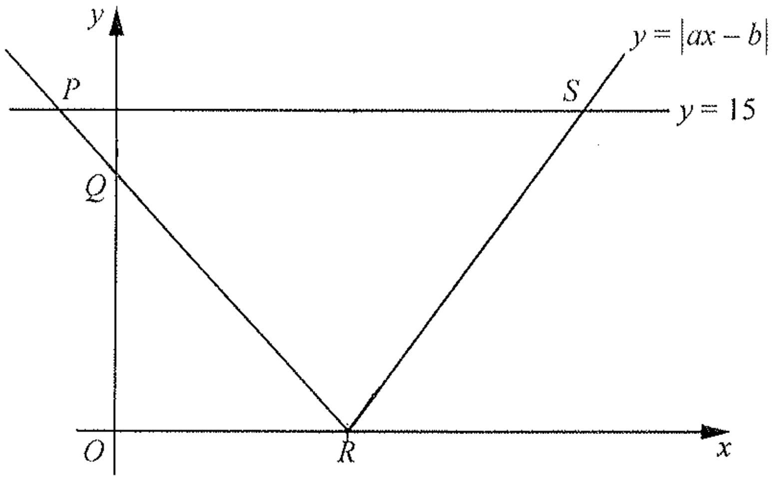

The diagram shows the line $y = 15$ and the graph of $y = |ax - b|$, where a and b are positive constants. The graph crosses the line $y = 15$ at the points P and S, crosses the y-axis at Q and meets the x-axis at R.

(i) Given that the x-coordinate of P is -2 and that $PQ : QR = 1 : 4$, find, with full explanation, the x-coordinate of S. [4]

(ii) Find the value of a and of b. [3]

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