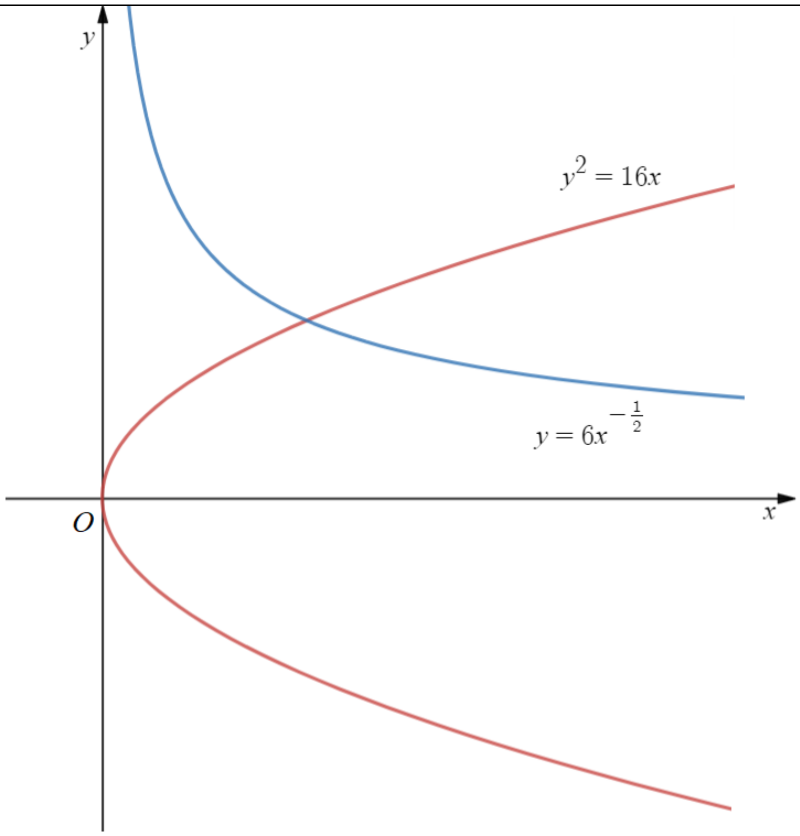
Suggested Solutions of 2018 Nov O Level Additional Mathematics Paper 1 Syllabus 4047

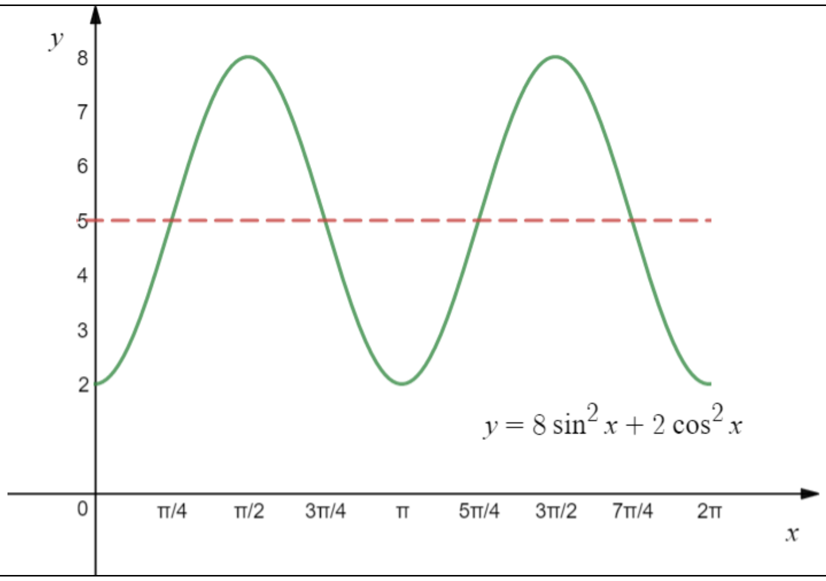
1	$\sqrt{125^x}=\dfrac{5^{1-x}}{25}$ $5^{\frac{3x}{2}}=5^{1-x-2}$ $3x=-2x-2\Rightarrow x=-\dfrac{2}{5}$ $\therefore\ \sqrt{125^x}=\sqrt{125^{-\frac{2}{5}}}$ $=\dfrac{\sqrt[5]{25}}{5}$ or 0.381 (3 s.f.)	 [1] [2] [1]
2(i)	$A+B+C=180°$ $\tan(A+B+C)=\tan 180°$ $\dfrac{\tan(A+B)+\tan C}{1-\tan(A+B)\tan C}=0$ $\tan(A+B)+\tan C=0$ $\therefore \tan C=-\tan(A+B)$ (shown)	 [1] [1]
2(ii)	$\therefore \tan C$ $=-\tan(A+B)$ $=-\tan(45°+60°)$ $=-\dfrac{\tan 45°+\tan 60°}{1-\tan 45°\tan 60°}$ $=-\dfrac{1+\sqrt{3}}{1-\sqrt{3}}\times\dfrac{1+\sqrt{3}}{1+\sqrt{3}}$ $=-\dfrac{1+2\sqrt{3}+3}{1-3}$ $=2+\sqrt{3}$ where $a=2,\ b=3$	 [1] [1] [1]
3	Let $\dfrac{7x^2-12x+17}{(2x-1)(x^2+4)}=\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+4}$ for some constants $A,\ B$ and C. $7x^2-12x+17=A(x^2+4)+(Bx+C)(2x-1)$ Let $x=\dfrac{1}{2}$: $7\left(\dfrac{1}{2}\right)^2-12\left(\dfrac{1}{2}\right)+17=A\left[\left(\dfrac{1}{2}\right)^2+4\right]$ $\dfrac{17}{4}A=\dfrac{51}{4}\Rightarrow A=3$ Let $x=0$: $7(0)^2-12(0)+17=3\left[(0)^2+4\right]+(0+C)(-1)$ $12-C=17\Rightarrow C=-5$	[1] [1] [1] [1]

	Let $x=1$: $7(1)^2-12(1)+17=3\left[(1)^2+4\right]+(B-5)(2-1)$ $B-5+15=12 \Rightarrow B=2$ $\therefore \dfrac{7x^2-12x+17}{(2x-1)(x^2+4)}=\dfrac{3}{2x-1}+\dfrac{2x-5}{x^2+4}$	[1] [1]
4(a)	Since $3+2\sqrt{5}$ is a root of $x^2+ax+b=0$, $\left(3+2\sqrt{5}\right)^2+a\left(3+2\sqrt{5}\right)+b=0$ $9+12\sqrt{5}+20+3a+2a\sqrt{5}+b=0$ $3a+b+29+(12+2a)\sqrt{5}=0$ Comparing rational parts of the equation, $3a+b+29=0 \Rightarrow b=-3a-29$ Comparing irrational parts of the equation, $12+2a=0 \Rightarrow a=-6$ $\Rightarrow b=-3(-6)-29=-11$ $\therefore a=-6,\ b=-11$ Alternative 1: Roots of the equation $x^2+ax+b=0$ are $\dfrac{-a\pm\sqrt{a^2-4b}}{2}$ -(1). $3+2\sqrt{5}=\dfrac{6+4\sqrt{5}}{2}=\dfrac{6+\sqrt{80}}{2}$ -(2) Comparing -(1) and -(2), $-a=6 \Rightarrow \therefore a=-6$. $a^2-4b=80$ $(-6)^2-4b=80$ $4b=-44 \Rightarrow \therefore b=-11$ Alternative 2: Since *a* and *b* are integers, the other distinct root of the equation $x^2+ax+b=0$ is $3-2\sqrt{5}$. $\Rightarrow$ Sum of roots $=3+2\sqrt{5}+3-2\sqrt{5}$ $=6$ $\Rightarrow$ Product of roots $=\left(3+2\sqrt{5}\right)\left(3-2\sqrt{5}\right)$ $=9-20$ $=-11$ Equation is $x^2-6x-11=0$. Comparing with $x^2+ax+b=0$, $\therefore a=-6,\ b=-11$.	[1] [2]

4(b)	$\therefore$ Breadth of rectangle $=\dfrac{24+\sqrt{48}}{6+\sqrt{12}}$ $=\dfrac{24+4\sqrt{3}}{6+2\sqrt{3}}\times\dfrac{6-2\sqrt{3}}{6-2\sqrt{3}}$ $=\dfrac{144-48\sqrt{3}+24\sqrt{3}-24}{36-12}$ $=\dfrac{120-24\sqrt{3}}{24}$ $=5-\sqrt{3}$ where $c=5,\ d=-1$	[1] [1] [1]
5(i)		[1] correct shape [1] drawn within $x \geq 0$ [1] graph, axes labelled

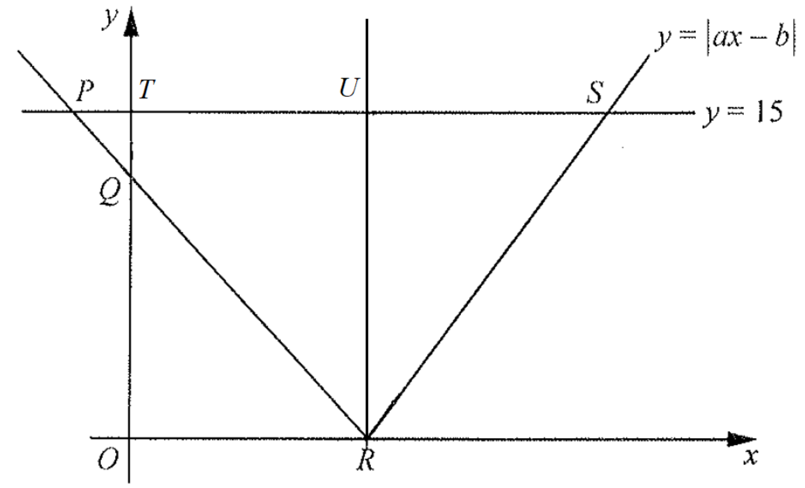
5(ii)	$y = 6x^{-\frac{1}{2}}$ -(1) $y^2 = 16x$ -(2) Substitute -(1) into -(2), $\left(6x^{-\frac{1}{2}}\right)^2 = 16x$ $36x^{-1} = 16x$ $x^2 = \frac{9}{4}$ $x = \frac{3}{2}$ or $-\frac{3}{2}$ (N.A. $\because x \geq 0$) When $x = \frac{3}{2}$, $y = 6\left(\frac{3}{2}\right)^{-\frac{1}{2}}$ $= \frac{6\sqrt{2}}{\sqrt{3}}$ $= 2\sqrt{6}$ or 4.90 (3 s.f.) $\therefore$ Coordinates of the point of intersection is $\left(\frac{3}{2}, 2\sqrt{6}\right)$.	[1] [1]
6(i)	$\therefore \log_3 x + \log_9 x$ $= \frac{\lg x}{\lg 3} + \frac{\lg x}{\lg 9}$ $= \frac{\lg x}{\lg 3} + \frac{\lg x}{2\lg 3}$ $= \frac{2\lg x + \lg x}{2\lg 3}$ $= \frac{3\lg x}{2\lg 3}$ (shown)	[1] [1] [1]
6(ii)	$\log_3 x + \log_9 x = 4$ $\frac{3\lg x}{2\lg 3} = 4$ $\lg x = \frac{8\lg 3}{3}$ $\therefore x = 10^{\frac{8\lg 3}{3}}$ $= 3^{\frac{8}{3}}$ $= 9\sqrt[3]{9}$ or 18.7 (3 s.f.)	[1] [1]

7(i)	Let $x=9$, $V=\frac{1}{3}\pi(9)^2(36-9)$ $=729\pi$ $\therefore$ Time taken $=\frac{729\pi}{18\pi}$ $=40\frac{1}{2}$ seconds	 [1] [1] [1]
7(ii)	$V=\frac{1}{3}\pi x^2(36-x)$ $=12\pi x^2-\frac{1}{3}\pi x^3$ $\frac{dV}{dx}=24\pi x-\pi x^2$ When $x=9$, $\frac{dV}{dx}=24\pi(9)-\pi(9)^2=135\pi$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ $18\pi=135\pi\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{18\pi}{135\pi}=\frac{2}{15}$ $\therefore$ Rate of change of the depth of water is $\frac{2}{15}$ cm per second.	 [1] [1] [1] [1]
8(i)	$\therefore 8\sin^2 x+2\cos^2 x$ $=8(1-\cos^2 x)+2\cos^2 x$ $=8-6\cos^2 x$ $=8-6\left(\frac{1+\cos 2x}{2}\right)$ $=8-3-3\cos 2x$ $=5-3\cos 2x$ where $a=5$, $b=-3$ (shown)	 [1] [1]
8(ii)	$\therefore$ Period $=\frac{2\pi}{2}=\pi$ radians $\therefore$ Amplitude $=3$	[1] [1]

8(iii)		[1] correct shape [1] correct amplitude, min. and max. y values [1] correct period, drawn within $0 \le x \le 2\pi$
9(i)	Gradient of AC $= \frac{10-0}{0-5}$ $= -2$ Gradient of OB $= \frac{-1}{-2}$ $\because$ AC and OB are $\perp$ diagonals of kite $= \frac{1}{2}$ Since OB passes through origin, $\therefore$ equation of OB is $y = \frac{1}{2}x$.	[1] [1] [1]
9(ii)	Let M be the midpoint of OB, then AC intersects OB at M $\because$ symmetrical property of kite Equation of AC is $y = -2x + 10$ -(1) OB: $y = \frac{1}{2}x$ -(2) Substitute -(1) into -(2), $-2x + 10 = \frac{1}{2}x$ $\frac{5}{2}x = 10 \Rightarrow x = 4$ When $x = 4$, $y = \frac{1}{2}(4) = 2$. $\Rightarrow M$ is $(4,2)$. Let B be (x_B, y_B), then M is $\left(\frac{x_B+0}{2}, \frac{y_B+0}{2}\right)$. $\frac{x_B}{2} = 4 \Rightarrow x_B = 8$, $\frac{y_B}{2} = 2 \Rightarrow y_B = 4$ $\therefore B$ is $(8,4)$.	[1] [1] [1] [1]

10(i)	$3x+2\pi r=20$ $2\pi r=20-3x$ $\therefore r=\dfrac{20-3x}{2\pi}$	[1]
10(ii)	$\therefore$ Total area, A $=\dfrac{1}{2}(x)(x)(\sin 60°)+\pi r^2$ $\quad\because$ interior $\angle$ of equilateral $\Delta=60°$ $=\dfrac{x^2}{2}\left(\dfrac{\sqrt{3}}{2}\right)+\pi\left(\dfrac{20-3x}{2\pi}\right)^2$ $=\dfrac{\sqrt{3}x^2}{4}+\dfrac{(20-3x)^2}{4\pi}$ $=\dfrac{\sqrt{3}\pi x^2+(20-3x)^2}{4\pi}$ (shown)	[1] [1] [1]
10(iii)	$A=\dfrac{\sqrt{3}\pi x^2+(20-3x)^2}{4\pi}$ $=\dfrac{\sqrt{3}}{4}x^2+\dfrac{400-120x+9x^2}{4\pi}$ $=\dfrac{\sqrt{3}}{4}x^2+\dfrac{9}{4\pi}x^2-\dfrac{30}{\pi}x+\dfrac{100}{\pi}$ $=\left(\dfrac{\sqrt{3}}{4}+\dfrac{9}{4\pi}\right)x^2-\dfrac{30}{\pi}x+\dfrac{100}{\pi}$ $\dfrac{dA}{dx}=\left(\dfrac{\sqrt{3}}{2}+\dfrac{9}{2\pi}\right)x-\dfrac{30}{\pi}$ At stationary values of A, $\dfrac{dA}{dx}=0$. $\left(\dfrac{\sqrt{3}}{2}+\dfrac{9}{2\pi}\right)x-\dfrac{30}{\pi}=0$ $\therefore x=\dfrac{30}{\pi}\times\dfrac{2\pi}{\sqrt{3}\pi+9}$ $=\dfrac{60}{\sqrt{3}\pi+9}$ or 4.15 (3 s.f.)	[1] [1] [1] [1]
10(iv)	$\dfrac{d^2A}{dx^2}=\dfrac{\sqrt{3}}{2}+\dfrac{9}{2\pi}=2.30$ (3 s.f.) Since $\dfrac{d^2A}{dx^2}>0$, the nature of this stationary $x=4.15$ is a minimum. $\therefore$ The gardener might be disappointed because this stationary $x=4.15$ yields a minimum value of the total area of his two flower beds. OR To maximize the total area, he can only maximize the area of either one of his flower beds while the other cannot exist.	[1] [1]

11(i)	$\mathrm{f}'(x)=a+\dfrac{b}{2x-3}$ $\dfrac{10x-9}{2x-3}=a+\dfrac{b}{2x-3}$ $10x-9=a(2x-3)+b$ $10x-9=2ax-3a+b$ Comparing both sides, $2a=10\Rightarrow a=5$ $-3(5)+b=-9\Rightarrow b=6$ $\therefore \mathrm{f}'(x)=5+\dfrac{6}{2x-3}$ where $a=5,\ b=6$	[1] [1]
11(ii)	For all real $x>\dfrac{3}{2},\ 2x-3>0$ $\Rightarrow 5+\dfrac{6}{2x-3}>0\Rightarrow \mathrm{f}'(x)>0$ $\therefore\ \mathrm{f}(x)$ is an increasing function.	[1]
11(iii)	$\mathrm{f}'(x)=5+6(2x-3)^{-1}$ $\mathrm{f}''(x)=-6(2x-3)^{-2}(2)$ $=-\dfrac{12}{(2x-3)^2}$ Since $(2x-3)^2>0$, $-\dfrac{12}{(2x-3)^2}<0\Rightarrow \mathrm{f}''(x)<0$ for all real $x>\dfrac{3}{2}$ $\therefore\ \mathrm{f}'(x)$ is an decreasing function.	[1] [1] [2]
11(iv)	$\mathrm{f}(x)$ $=\int \mathrm{f}'(x)\ \mathrm{d}x$ $=\int 5+\dfrac{6}{2x-3}\ \mathrm{d}x$ $=5x+\dfrac{6\ln(2x-3)}{2}+c$ $=5x+3\ln(2x-3)+c$ for some real constant c Given $\mathrm{f}(2)=8$, $5(2)+3\ln(4-3)+c=8\Rightarrow c=-2$ $\therefore \mathrm{f}(x)=5x+3\ln(2x-3)-2$	[1] [1] [1] [1]

12(i)	Let U be on the line $y = 15$ vertically above R, then U is the midpoint of PS $\because$ modulus graph is symmetrical about $x = x_R$, where x_R is the x-coordinate of R Let T be the point $(0,15)$  $\angle PTQ = \angle PUR = 90°$ ($TQ \parallel UR$, and $TQ, UR \perp x$-axis) $\angle QPT = \angle RPU$ (common $\angle$) By AA property, ΔPQT is similar to ΔPRU.	[2]
	$\Rightarrow \dfrac{PU}{PT} = \dfrac{PR}{PQ}$ $\dfrac{PU}{PT} = \dfrac{PQ+QR}{PQ} = \dfrac{1+4}{1} = 5$	[1]
	Since x-coordinate of P is -2, $PT = 2$. $\Rightarrow PU = 5 \times 2 = 10$ Since U is the midpoint of PS, $PU = US$. $\therefore$ x-coordinate of S $= -2 + 10 + 10$ $= 18$	[1]
12(ii)	S is $(18,15)$. $x_R = \dfrac{-2+18}{2}$ (U is midpoint of PS) $= 8$ R is $(8,0)$.	[1]
	Gradient of RS $= \dfrac{15-0}{18-8}$ $= \dfrac{3}{2}$ Since $a > 0$, $a = \dfrac{3}{2}$.	[1]

	At $R(8,0)$, $\left\lvert\frac{3}{2}(8)-b\right\rvert=0$ $12-b=0 \Rightarrow b=12$ $\therefore a=\frac{3}{2},\ b=12$	[1]