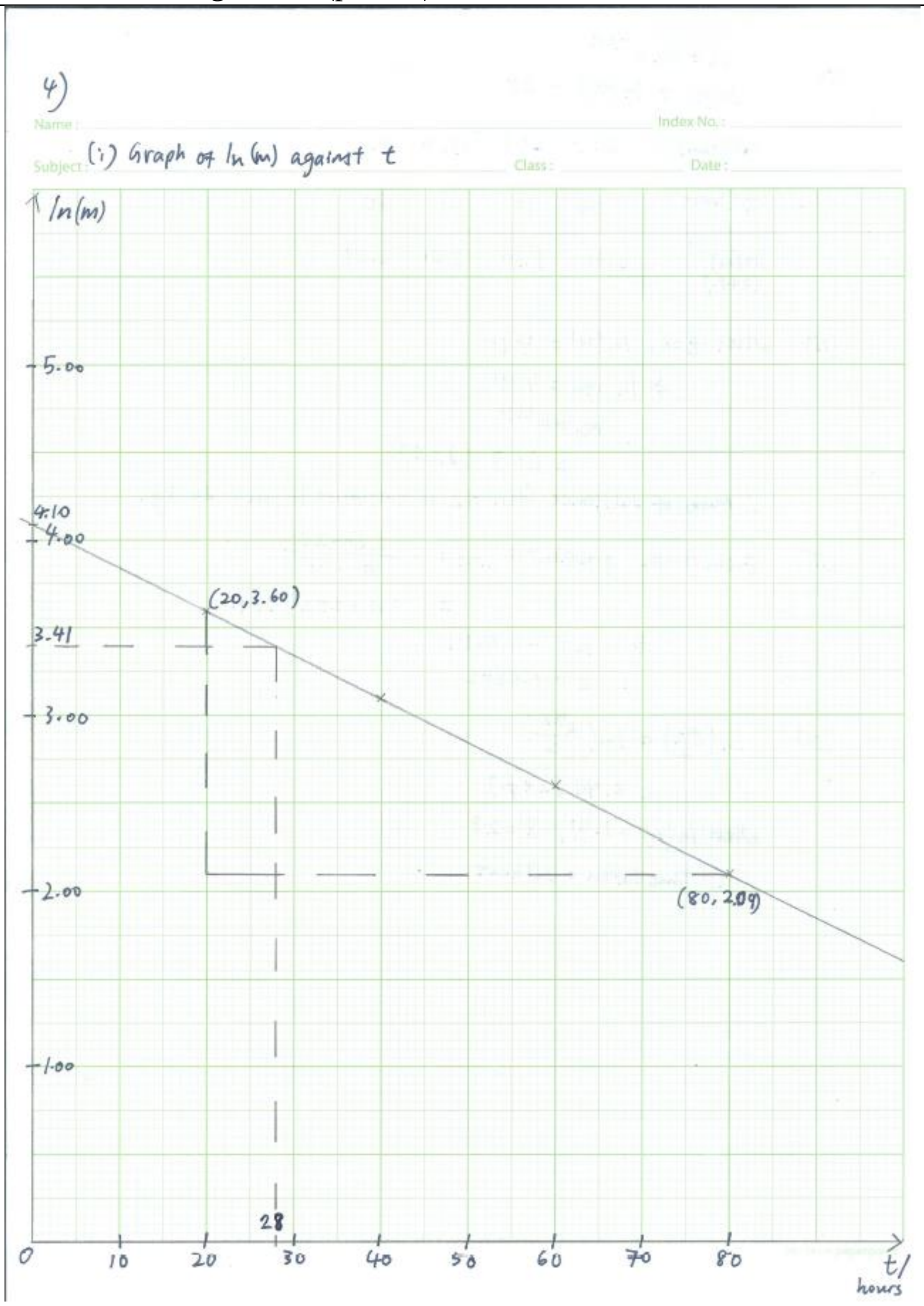
Suggested Solutions of 2018 Nov O Level Additional Mathematics Paper 2 Syllabus 4047

1(i)	$x^2+3x+5=0$ $\alpha+\beta=-3$ $\alpha\beta=5$ $\therefore(\alpha+1)(\beta+1)$ $=\alpha\beta+\alpha+\beta+1$ $=5+(-3)+1$ $=3$ (shown)	[1] [1] [1]
1(ii)	$\frac{2}{\alpha+1}+\frac{2}{\beta+1}=\frac{2\beta+2+2\alpha+2}{(\alpha+1)(\beta+1)}$ $=\frac{2(-3)+4}{3}$ $=-\frac{2}{3}$ $\left(\frac{2}{\alpha+1}\right)\left(\frac{2}{\beta+1}\right)=\frac{4}{(\alpha+1)(\beta+1)}$ $=\frac{4}{3}$ $\therefore$ Quadratic equation is $x^2+\frac{2}{3}x+\frac{4}{3}=0$ or $3x^2+2x+4=0$.	[1] [1] [1] [1]
2	$(1-4x)(2+ax)^6=(1-4x)\left[(2)^6+\binom{6}{1}(2)^5(ax)+\binom{6}{2}(2)^4(ax)^2+...\right]$ $=(1-4x)(64+192ax+240a^2x^2+...)$ $=64+192ax+240a^2x^2-256x-768ax^2+...$ $=64+(192a-256)x+(240a^2-768a)x^2+...$ Comparing coefficients of *x*, $192a-256=-160$ $192a=96$ $\therefore a=\frac{1}{2}$ Comparing coefficients of x^2, $b=240a^2-768a$ $=240\left(\frac{1}{2}\right)^2-768\left(\frac{1}{2}\right)$ $=-324$ $\therefore b=-324$	[2] [1] [1] [1] [1] [1]
3	$\angle APB=\angle PCQ$ (Alternate Segment Theorem) $\angle APB+\angle BPQ=\angle APQ$ $=\angle AQP$ ($AP=AQ$, base $\angle$s of isosceles Δ)	[1] [1]

	$\angle AQP = \angle PCQ + \angle CPQ$ (exterior $\angle$ of Δ = sum of interior opp $\angle$s) $\Rightarrow \angle APB + \angle BPQ = \angle PCQ + \angle CPQ$ Since $\angle APB = \angle PCQ$, then $\angle BPQ = \angle CPQ$. $\therefore PQ$ bisects angle BPC (proven).	[2] [1]
4(i)		[1] correctly labelled axes [1] points plotted correctly [1] straight line drawn through vertical axis and all 4 points

4(i)

$m = m_0 e^{-kt}$

$\ln(m) = \ln(m_0) - kt$

m (grams)	36.4	22.1	13.4	8.1
t (hours)	20	40	60	80
ln(m) (2d.p.)	3.60	3.10	2.60	2.09

4(ii)

When $t=0$, $\ln(m) = 4.10$ [1]

$\Rightarrow \ln m_0 = 4.10$

$m_0 = e^{4.10}$

$= 60.3$ (3s.f.)

$\therefore$ Mass of substance when the observations began $= 60.3g$. [1]

4(iii)

From graph, gradient of line $= \dfrac{3.60-2.09}{20-80}$

$= -0.0252$ (3s.f.) [1]

$\Rightarrow -k = -0.0252$

$\therefore k = 0.0252$ [1]

4(iv)

$\ln\left(\frac{m_0}{2}\right) = \ln\left(\frac{e^{4.10}}{2}\right)$

$= 3.41$ (2d.p.) [1]

When $\ln(m) = 3.41$, $t = 28$. [1]

$\therefore$ Time taken $= 28$ hours.

5(i)

$\dfrac{BC}{AB} = \tan\theta$

$\therefore BC = AB\tan\theta$

$= 800\tan\theta$ [1]

$\therefore CD = BD - BC$

$= 1200 - 800\tan\theta$ [1]

5(ii)

$\angle CED = \angle ABC = 90°$ (given)

$\angle DCE = \angle ACB$ (vertically opp $\angle$s)

$\Rightarrow \angle CDE = \angle BAC = \theta$ (angle sum of ΔABC and ΔCDE)

$\dfrac{DE}{CD} = \cos\angle CDE = \cos\theta$ [1]

$DE = CD\cos\theta$

$= (1200 - 800\tan\theta)\cos\theta$

$= 1200\cos\theta - 800\tan\theta\cos\theta$

$= 1200\cos\theta - 800\sin\theta$ (shown) where $a = 1200$, $b = -800$ [2]

5(iii)

$DE = 1200\cos\theta - 800\sin\theta$

$= \sqrt{1200^2 + 800^2}\cos\left(\theta + \tan^{-1}\dfrac{800}{1200}\right)$ [2]

$= 400\sqrt{13}\cos(\theta + 33.69°)$ [1]

where $R = 400\sqrt{13}$ or 1442 (4 s.f.), $\alpha = 33.69°$ (2 d.p.)

	Since $DE = 200$, $400\sqrt{13}\cos(\theta + 33.69°) = 200$ $\cos(\theta + 33.69°) = \dfrac{200}{400\sqrt{13}}$ $\theta + 33.69° = \cos^{-1}\left(\dfrac{1}{2\sqrt{13}}\right)$ $= 82.03°$ (2 d.p.) $\therefore \theta = 82.03° - 33.69°$ $= 48.3°$ (1 d.p.)	[1] [1]
6(i)	$\dfrac{d}{dx}(x\cos x) = \cos x + x(-\sin x)$ $= \cos x - x\sin x$	[2]
6(ii)	$\int \cos x - x\sin x \, dx = x\cos x + c_1$ for some constant c_1 $\int \cos x \, dx - \int x\sin x \, dx = x\cos x + c_1$ $\sin x + c_2 - x\cos x - c_1 = \int x\sin x \, dx$ for some constant c_2 $\therefore \int x\sin x \, dx = \sin x - x\cos x + c$ where $c = c_2 - c_1$	[1] [1]
6(iii)	$\dfrac{d}{dx}(x^2\sin x) = 2x\sin x + x^2\cos x$ $= x(2\sin x - x\cos x)$ $\int 2x\sin x + x^2\cos x \, dx = x^2\sin x + c_1$ for some constant c_1 $2\int x\sin x \, dx + \int x^2\cos x \, dx = x^2\sin x + c_1$ $2\sin x - 2x\cos x + c_2 + \int x^2\cos x \, dx = x^2\sin x + c_1$ for some constant c_2 $\therefore \int x^2\cos x \, dx = x^2\sin x + 2x\cos x - 2\sin x + c$ where $c = c_1 - c_2$	[2] [2] [2]
7(i)	$d = 840\left(1 - e^{-\frac{t}{80}}\right) - 2t$ Let v denote speed. $v = \dfrac{d(d)}{dt}$ $= 840\left(-\dfrac{1}{80}\right)\left(-e^{-\frac{t}{80}}\right) - 2$ $= \dfrac{21}{2}e^{-\frac{t}{80}} - 2$ When $t = 10$, $v = \dfrac{21}{2}e^{-\frac{10}{80}} - 2 = 7.27$ (3 s.f.) $\therefore$ Girl's speed $= 7.27$ m/s Let a denote acceleration.	[2] [1]

	$a = \frac{dv}{dt}$ $= \left(-\frac{1}{80}\right)\left(\frac{21}{2}\right)e^{-\frac{t}{80}}$ $= -\frac{21}{160}e^{-\frac{t}{80}}$ When $t = 10$, $a = -\frac{21}{160}e^{-\frac{10}{80}} = -0.116$ (3 s.f.) $\therefore$ Girl's acceleration $= -0.116$ m/s^2	[2] [1]
7(ii)	The acceleration has a negative sign which means that the girl is decelerating or reducing her cycling speed.	[1]
7(iii)	When $v = 1.5$, $\frac{21}{2}e^{-\frac{t}{80}} - 2 = 1.5$ $e^{-\frac{t}{80}} = \frac{1}{3}$ $\ln e^{-\frac{t}{80}} = \ln\frac{1}{3}$ $-\frac{t}{80} = -\ln 3$ $t = 80\ln 3$ When $t = 80\ln 3$, $d = 840\left(1 - e^{-\frac{80\ln 3}{80}}\right) - 2(80\ln 3)$ $= 560 - 160\ln 3$ $= 384.2$ (4 s.f.) $500 - 384.2 = 115.8$ $= 116$ (3 s.f.) $\therefore$ Distance she has to push her bicycle $= 116$ m	[2] [1] [1]
8(i)	Let $x + 2 = 0 \Rightarrow x = -2$ By Remainder Theorem, $p(-2) = 2(-2)^3 + 5(-2)^2 - 18$ $= -14$ $\therefore$ Remainder is -14.	[1] [1]
8(ii)	Let $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$ By Remainder Theorem, $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 + 5\left(\frac{3}{2}\right)^2 - 18$ $= 0$ Since remainder is 0 when $p(x)$ is divided by $2x - 3$, then by Factor Theorem $2x - 3$ is a factor of $p(x)$.	[1] [1]

8(iii)	$\text{p}(x)=2x^3+5x^2-18$ $=(2x-3)(ax^2+bx+c)$ for some constants $a,\ b$ and c Comparing coefficients of x^3, $2a=2\Rightarrow a=1$ Comparing constant terms, $-3c=-18\Rightarrow c=6$ Comparing coefficients of x, $2c-3b=0$ $3b=2(6)=12\Rightarrow b=4$ $\text{p}(x)=(2x-3)(x^2+4x+6)$ When $\text{p}(x)=0$, $(2x-3)(x^2+4x+6)=0$ $2x-3=0$ or $x^2+4x+6=0$ $x=\frac{3}{2}$ discriminant $=(4)^2-4(1)(6)$ $=-8$ Since discriminant $=-8<0$, there are no real roots for $x^2+4x+6=0$. $\therefore \text{p}(x)=0$ has only one real root which is $x=\frac{3}{2}$.	[2] [1] [1]
8(iv)	$2^{3y+1}+5(2^{2y})=18$ $(2^{3y})(2)+5(2^y)^2=18$ $2(2^y)^3+5(2^y)^2-18=0$ Let $x=2^y\Rightarrow 2x^3+5x^2-18=0$ Since $x=\frac{3}{2}$ is the only real root when $2x^3+5x^2-18=0$, $2^y=\frac{3}{2}$. $\therefore y=\frac{\lg\left(\frac{3}{2}\right)}{\lg 2}$ $=0.585$ (3 s.f.)	[1] [1] [1]
9(i)	When $k=5$, $y=2x^2+7x+5$ Substitute $y=19x-13$ into $y=2x^2+7x+5$, $2x^2+7x+5=19x-13$ $2x^2-12x+18=0$ $x^2-6x+9=0$ Since discriminant $=(6)^2-4(1)(9)=0$, the equation has real and equal roots. There is only one point of intersection between the line and the curve. $\therefore$ $y=19x-13$ is a tangent to the curve (shown). $x^2-6x+9=0$ $\Rightarrow(x-3)^2=0$ $x-3=0$ (repeat) $x=3$	[1] [1]

	Substitute $x=3$ into $y=19x-13$, $y=19(3)-13$ $=44$ $\therefore$ Point of contact is $(3,44)$.	[1]
9(ii)	Since *y* cannot be negative, the curve *y* must lie above $y=0$ or $y=0$ must be tangent to the curve. $\Rightarrow$ discriminant ≤ 0 $(k+2)^2-4(2)(k)\le 0$ $k^2+4k+4-8k\le 0$ $k^2-4k+4\le 0$ $(k-2)^2\le 0$ Since $(k-2)^2\ge 0$ for all real values of *k*, the only value of *k* for $(k-2)^2\le 0$ is when $k-2=0\Rightarrow\therefore k=2$ <u>Alternative:</u> $y=2x^2+(k+2)x+k$ $=2\left(x^2+\frac{k+2}{2}x\right)+k$ $=2\left[x^2+\frac{k+2}{2}x+\left(\frac{k+2}{4}\right)^2-\left(\frac{k+2}{4}\right)^2\right]+k$ $=2\left(x+\frac{k+2}{2}\right)^2-\frac{(k+2)^2}{8}+k$ Since *y* cannot be negative, $y\ge 0$ and the minimum value of *y* is at least zero. $\Rightarrow -\frac{(k+2)^2}{8}+k\ge 0$ $-\frac{(k+2)^2}{8}+k\ge 0$ $-k^2-4k-4+8k\ge 0$ $k^2-4k+4\le 0$ $(k-2)^2\le 0$ Since $(k-2)^2\ge 0$ for all real values of *k*, the only value of *k* for $(k-2)^2\le 0$ is when $k-2=0\Rightarrow\therefore k=2$.	[1] [1] [2]
10(i)	$y=2\sqrt{7-3x}$ $\frac{dy}{dx}=2\left(\frac{1}{2}\right)(7-3x)^{-\frac{1}{2}}(-3)$ $=-\frac{3}{\sqrt{7-3x}}$	[1] [1]

	At P, $\frac{dy}{dx}=-\frac{3}{\sqrt{7-3k}}$ Gradient of tangent at P is $-\frac{3}{\sqrt{7-3k}}$ $\Rightarrow$ Gradient of normal at P $=-1\div-\frac{3}{\sqrt{7-3k}}$ $=\frac{\sqrt{7-3k}}{3}$ $\frac{2\sqrt{7-3k}-0}{k-(-5)}=\frac{\sqrt{7-3k}}{3}$ (Gradient of NP) Since $\sqrt{7-3k}\neq 0 \because k\neq\frac{7}{3}$, $\frac{2}{k+5}=\frac{1}{3}$ $k+5=6$ $\therefore k=1$ (proven)	[1] [2]
10(ii)	Let x_T be the x-coordinate of $T \Rightarrow T$ is $(x_T,0)$ Gradient of the tangent to the curve at P is $-\frac{3}{\sqrt{7-3(1)}}=-\frac{3}{2}$ $\Rightarrow \frac{2\sqrt{7-3(1)}-0}{(1)-x_T}=-\frac{3}{2}$ (Gradient of TP) $\frac{4}{1-x_T}=-\frac{3}{2}$ $1-x_T=-\frac{8}{3}$ $x_T=1+\frac{8}{3}$ $=\frac{11}{3}$ $\therefore$ x-coordinate of T is $\frac{11}{3}$ (shown).	[1] [1]

10(iii)	Let F be the foot of perpendicular from P to the x-axis. $\Rightarrow F$ is $(1,0)$ y-coordinate of $P = 2\sqrt{7-3(1)} = 4 \Rightarrow PF = 4$ $FT = x_T - 1$ $= \frac{11}{3} - 1$ $= \frac{8}{3}$ Area bounded by the curve, the line $x=1$ and the x-axis $= \int_1^{\frac{7}{3}} 2\sqrt{7-3x}\ \mathrm{d}x$ $= \left[\frac{2(7-3x)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(-3)}\right]_1^{\frac{7}{3}}$ $= \left[-\frac{4}{9}\sqrt{(7-3x)^3}\right]_1^{\frac{7}{3}}$ $= 0 - \left(-\frac{32}{9}\right)$ $= 3\frac{5}{9}$ units^2 $\therefore$ Area of shaded region PXT $=$ Area of $\Delta PFT - 3\frac{5}{9}$ $= \frac{1}{2} \times FT \times PF - 3\frac{5}{9}$ $= \frac{1}{2}\left(\frac{8}{3}\right)(4) - 3\frac{5}{9}$ $= 1\frac{7}{9}$ units^2	[2] [1] [1] [1]
11(i)	Gradient of $AB = \frac{8-4}{9-1}$ $= \frac{1}{2}$ Gradient of $BC = \frac{12-8}{7-9}$ $= -2$ Gradient of $AB \times$ Gradient of $BC = \frac{1}{2} \times (-2)$ $= -1$ $\Rightarrow AB \perp BC$ and $\therefore$ angle $ABC = 90°$ (shown)	[1] [2]

11(ii)	*A*, *B* and *C* lie on a circle with diameter *AC* because *ABC* is a right-angled triangle in a semicircle with side *AC* opposite of the right angle (angle $ABC = 90°$).	[1]
11(iii)	Since *AC* is the diameter, centre of circle is the midpoint of *AC*. Centre of circle is $\left(\frac{1+7}{2}, \frac{4+12}{2}\right) = (4,8)$ Radius of circle $= \frac{1}{2} \times AC$ $= \frac{1}{2} \times \sqrt{(7-1)^2 + (12-4)^2}$ $= \frac{\sqrt{100}}{2}$ $= 5$ units $\therefore$ Equation of circle is $(x-4)^2 + (y-8)^2 = 25$.	[1] [1] [1]
11(iv)	Let the centre of the circle be *O*. $\Rightarrow O$ is $(4,8)$ Gradient of radius *OB* $= \frac{8-8}{9-4}$ $= 0$ Since tangent $\perp$ radius, gradient of tangent at *B* is $-\frac{1}{0}$ which is undefined. $\therefore$ The tangent to the circle at *B*, which is $x = 9$, is parallel to the *y*-axis.	[2]
11(v)	Gradient of radius *OC* $= \frac{12-8}{7-4}$ $= \frac{4}{3}$ Since tangent $\perp$ radius, gradient of the tangent at *C* $= -1 \div \frac{4}{3}$ $= -\frac{3}{4}$ $\Rightarrow y - 12 = -\frac{3}{4}(x-7)$ $y = -\frac{3}{4}x + \frac{21}{4} + 12$ $y = -\frac{3}{4}x + \frac{69}{4}$ $\therefore$ Equation of the tangent to the circle at *C* is $y = -\frac{3}{4}x + \frac{69}{4}$ or $4y + 3x = 69$.	 [1] [1]